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APPROXIMATE ANALYTICAL INVESTIGATION OF THE ELASTIC-PLASTIC BEHAVIOUR OF FIBROUS COMPOSITES. II. EXTERNAL LOADING

KLAUS HERRMANN, IVAN MIHOVSKY

Клаус Херман, Иван Миховски. ПРИБЛИЖЕННОЕ АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ УПРУГОПЛАСТИЧЕСКОГО ПОВЕДЕНИЯ ВОЛОКНИСТЫХ КОМПОЗИТОВ. II. ВНЕШНЕЕ НАГРУЖЕНИЕ.

Работа продолжает исследование композитов, рассмотренных в части I, посвященной их поведению в условиях термического нагружения. Здесь исследован случай чисто механического нагружения и точнее — продольного растяжения. Показано, что предложенный в части I подход ведет к надежным качественным и количественным заключениям и оценкам относительно поведения рассматриваемых композитов.

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The paper continues the investigation of the composites specified in Part I. While the latter part is devoted to the thermally induced response the present one deals with the purely mechanical problem of longitudinal extension. The approach developed in Part I is shown to lead to realistic (both qualitative and quantitative) predictions of the overall response of the composites considered.

INTRODUCTION

The basic aspects of the influence of the matrix plasticity on the overall thermomechanical response of the fibrous composites are considered in sufficient detail in the introductory section of Part I of the present study along with the specific features of the general approach developed in the latter.

In the present part the same class of fibrous composites is considered by the aid of the same models of the composite unit cell and the process of matrix plastification (Herrmann & Mihovsky [1, 2], cf. p. I). The loading is specified as longitudinal extension, which is a typical operational loading for fibrous composites. Therefore it is quite natural that their response in such load-bearing applications has been intensively studied in the past and that a good understanding of the overall characteristics of this response already exists nowadays.

Following Kelly [3] one may summarize that there are two stages in the behaviour of the considered composites. They reflect the initial purely elastic elongation of the fibres and the matrix, respectively, as well as the following plastic flow in the matrix. The transition to the second stage occurs when the matrix material starts yielding. This process begins at a value of the axial strain which is "a little less" than the yield strain of the matrix, and is complete at "a slightly larger" strain. The contribution of the matrix to the stress-strain curve in the second stage is negligible. The lower bound to this slope, as derived by Hill [4], is (in the notation introduced in p. I) $E_f r_f^2/r_m^2$.

This brief general description of the elastic-plastic response of the fibrous composites clearly indicates that the entire matrix plastification is a sudden phenomenon. Such an effect of a sudden entire matrix plastification is not involved in the thermally induced composite response, considered in p. I. The plastic zone size in the latter case has been shown to increase monotonically with progressive thermal loading. To clear up the response of the composites in the interval between the initial and the complete matrix plastification, respectively, as well as the very mechanism of the latter appears to be an interesting problem. In fact, this interval corresponds to a very small change of the axial strain from "a little less" to "a slightly larger" value, as stated by Kelly [3]. Thus, at fisrt sight, the details of the composite behaviour in this short interval do not seem to be of essential significance. But, from the view point of the influence of the matrix plasticity on the response (including the failure) of the composites, it is important to get a better understanding of this initial stage of matrix plastification. The real nature of this stage indicates that it is governed by specific mechanisms. Accordingly, one should expect that the latter may further contribute to the occurrence of specific trends of the development of the plastic deformation process in the completely plastified matrix. These trends are of interest with respect to the determination of the overall response of the composites, especially for the occurrence and the development of failure modes.

In this paper, the specific aspects of the matrix plastification process developing in a longitudinally extended fibrous composite are considered. Corresponding conclusions of both qualitative and quantitative character are derived by means of an approximate analytical version of the general approach already used in Part I. This version predicts an elastic-plastic response of the composite material which is consistent with the lower bound estimation obtained by Hill [4]. Certain general conclusions are derived in the closing section of the article with respect to the distinguishing features of the composite response under thermal and mechanical loading conditions. The specific influence of the matrix plasticity on the fracture resistance of the composites for both loading schemes as well as the significance of special structural defects are considered.

STATEMENT OF THE PROBLEM

The class of composites, the composite unit cell, and the mechanical properties of the fibre and the matrix materials, respectively, are the same as already specified in p. I. The loading is specified as longitudinal extension of the composite and therefore as axial extension of the unit cell. The lateral surface of the latter is traction free. Accordingly, the plane cross-sections hypothesis applies, the stress-strain field in the cell is axisymmetric, and the normal stresses in both the fibre and the matrix are principal ones and depend upon the radial coordinate r only. Further, the powers and the products of the ratios E_m/E_f and r_f/r_m are considered again as small quantities and, like in p. I, the final results are presented in forms, containing the principal terms only.

ELASTIC BEHAVIOUR AND ELASTIC-PLASTIC TRANSITION

In accordance with the known elastic solution of the problem (cf., for example, Ebert et al. [5]) the stresses in the matrix and the fibre are of the same form as in eqns (1), p. I, but with a new value of the constant C. This value reads

(1) $$C = \varepsilon_z \Delta\nu r_f^2,$$

where the notation $\Delta\nu$ means

(2) $$\Delta\nu = \nu_m - \nu_f.$$

Moreover, the relation

(3) $$\Delta\nu > 0$$

applies for the commonly used fibrous composites and is assumed to be valid in the following considerations. It provides, in fact, the occurrence of compressive radial stresses at the fibre-matrix interface, i.e. the well-known shrinkage effect. Further, by considering the equilibrium condition for the axial forces, the elastic stress distribution from eqns (1), p. I, together with the new C-value from eqn (1) now implies the relation

(4) $$\sigma_z^c = \varepsilon_z E_m(1 + E_c),$$

where E_c is the same as in eqn (3), p. I, while

$$\sigma_z^c = P/\pi r_m^2 \tag{5}$$

is the axial composite stress, induced in the unit cell by the applied axial tensile force P. Eqn (4) represents the well-known "rule of mixtures" approximation of the linear elastic response of a composite.

Now, in accordance with the classical von Mises' yield condition, the matrix starts yielding at the fibre-matrix interface when the stress σ_z^c and the axial strain ε_z, respectively, achieve the values

$$\sigma_z^{c,pl} = \sigma_y(1+E_c)\left[1 - \frac{3}{2}\frac{(\Delta\nu)^2}{(1+\nu_m)^2}\right], \tag{6}$$

$$\varepsilon_z^{pl} = \frac{\sigma_y}{E_m}\left[1 - \frac{3}{2}\frac{(\Delta\nu)^2}{(1+\nu_m)^2}\right]. \tag{7}$$

Due to the smallness of $(\Delta\nu)^2$ one may really view the strain ε_z^{pl} of the initial matrix plastification, given in eqn (7), as "a little less" than the matrix yield strain $\varepsilon_y = \sigma_y/E_m$, as stated by Kelly [3]. In addition, a simple comparison with the thermal problem, considered in p. I, shows that in the present case the initial matrix plastification takes place at a much larger value of the ε_z-strain than it was stated for the corresponding ε_z^{sts}-value (cf. p. I, eqn (8)). On the contrary, the value of the radial stress at the fibre-matrix interface at the instant of initial plastification is much smaller than the corresponding stress value in the thermal problem.

It should be mentioned that in accordance with the sense of the quantity $\overset{*}{\varepsilon}{}_z^e$, involved in the matrix plastification model, its actual value should be expected to be close to the ε_z^{pl}-value or, respectively, to the value $\varepsilon_z^{sts,pl}$ in the thermal problem (cf. p. I). Therefore, the plastic behaviour of the matrix in the present case will be associated with a yield ellipse, which is of the same geometry as that in the thermal case (p. I, eqns (10), (11)) but with its center removed from the origin of the (σ_θ, σ_r)-plane along the line $\sigma_r = \sigma_\theta$ over a distance which is "a little less" than the length $\sigma_y/(1-2\nu_m)$ of its larger principal half-axis. In the thermal case the center of the yield ellipse almost coincides with the origin of the (σ_θ, σ_r)-plane. In the present problem it is its vertex $\omega = \pi$ (p. I, eqn (12)) that almost coincides with the same origin. These general observations will prove to be useful for the following analysis.

ANALYSIS OF THE ELASTIC-PLASTIC BEHAVIOUR

In accordance with the basic adoptions of the matrix plastification model (cf. p. I) one may immediately conclude that the series of equations given in p. I, namely eqns (9) — (18), should hold true in the present problem as well. Further, when keeping the structure of the thermal problem analysis one should consider as a next step the way, in which the shrinkage influences the behaviour of the angle ω_{r_f}, introduced in eqn (18), p. I. Generally speaking, the shrinkage is a desired effect in the load-bearing applications of the fibrous composites, since it prevents the occurrence of delamination phenomena in the latter. Practically, the shrinkage in the present problem results from the larger cross-sectional contraction of the matrix with respect to the fibre. Relation (3) simply proves the validity of this conclusion in the elastic range. Furthermore, the usual assumption of plastic incompressibility implies the natural conclusion that the progressive plastification of the matrix will effectively result in a further increase of Poisson's ratio in the matrix phase. The latter increase will then contribute to the further increase of the shrinkage as it is adopted in the matrix plastification model of Herrmann & Mihovsky [1]. In addition, one should accordingly accept that as in the thermal problem the progressive loading will cause monotonous increase of the angle ω_{r_f} within the interval defined by eqn (19), p. I. But by considering the u_r-continuity condition at the fibre-matrix interface (p. I, eqn (21)), it can be easily seen that this foregoing adoption is not realistic. Because in the present case the function $f(\omega_{r_f})$ has again negative values, whereas, in contrast to the thermal one, $d\varepsilon_z$ is positive due to the elongation of the cell. Therefore, the equation cited predicts negative $d\omega_{r_f}$-values, that means a decrease of ω_{r_f} and thus of the shrinkage with progressive loading, i.e. with increasing ε_z-strain. To clear up the reason for this inconsistency between the adoption mentioned above and the prediction of the u_r-continuity condition appears to be the first necessary step that distinguishes the present analysis from that of the thermal problem.

It should be recalled in this regard that the u_r-continuity condition, eqn (21), p. I, as well as the boundary condition, eqn (15), p. I, are derived under the assumption that the $\varepsilon_i^{e,sts}$- strain ($i = r, \theta$) are neligible (cf. the text following eqn (15), p. I). One may simply prove that, in fact, it is this assumption that leads to the inconsistency mentioned just above. Actually, in the present case, the cross-sectional elastic strain components in the plastic zone should not be neglected since, due to the relatively small stress concentration effect of the fibre (cf. the remarks following eqn (7)), the corresponding plastic strain components will be also small enough. Finally, by means of the procedure, described in sec. 5, p. I, and with the ε_θ^e-strain introduced into the u_r-continuity condition, the following relation holds true

(8) $$\Lambda_1 d\varepsilon_z = f_1(\omega_{r_f})d\omega_{r_f},$$

where

(9) $$\Lambda_1 = \frac{\sqrt{3}E_m}{4\sigma_y(1+\nu_m)(1+\alpha)\sin\Phi},$$

$$f_1(\omega_{r_f}) = \frac{\sin\left(\frac{\pi}{6} - \omega_{r_f}\right)\cos\omega_{r_f}}{\sin(\omega_{r_f} + \Phi) - 2\nu_f \sin\Phi \cos\omega_{r_f}}, \tag{10}$$

$$\alpha = \frac{(1+\nu_f)(1-2\nu_f)E_m}{(1+\nu_m)E_f}. \tag{11}$$

Thereby eqn (8) is to be further coupled with a corresponding interval within which the angle ω_{r_f} changes, as well as with an appropriate boundary condition. It can be proved that the angle ω_{R_c}, as defined by eqn (15), p. I (with the elastic strain neglected), will not apply to the present case. Thus, the account for the elastic strain, involved in eqn (8), requires a more accurate determination of the initial value of ω_{r_f}, i.e. of ω_{R_c}. The latter represents itself the value of ω_{r_f} at the instant of the occurence of the second plastic zone. As in the thermal case it can be assumed that up to this instant the transitional matrix plastification process (cf. p. I) does not affect substantially the linear elastic behaviour of the composite. In the present case this behaviour allows to be considered as satisfying the condition

$$(\sigma_r^{me} + \sigma_\theta^{me})|_{r=r_f} = 0, \tag{12}$$

since the left-hand side of eqn (12) is proportional to r_f^2/r_m^2 and the terms of this order of magnitude are, as adopted, neglected in the present analysis.

Moreover, since the value $\overset{*e}{\varepsilon}_z$ of the ε_z-strain, at which the second plastic zone occurs, should not differ substantially from the value of the ε_z^{pl}-strain (cf. eqn (7)), one may further accept that

$$\overset{*e}{\varepsilon}_z = \varepsilon_z^{pl}. \tag{13}$$

Regarding the yield ellipse (p. I, eqns (10), (11) now with $\overset{*e}{\varepsilon}_z$ from eqn (13)) eqns (12) and (13) simply prove that the stress state at the fibre-matrix interface, at which the second plastic zone occurs, corresponds to the intersection point of this ellipse and of the straight line defined by eqn (12). Accordingly, the value ω_{R_c}, which defines the position of this intersection point over the yield ellipse, can be determined by using eqns (10), p. I, as well as eqns (7) and (13), respectively

$$\omega_{R_c} = \arccos\left[-1 + \frac{3}{2}\frac{(\Delta\nu)^2}{(1+\nu_m)^2}\right]. \tag{14}$$

Due to the smallness of $(\Delta\nu)^2$ the value of ω_{R_c} is approximately π. The same statement is valid for the angle $\pi - \Phi$ (cf. p. I, eqn (13)). Now, in the framework of the model of the matrix plastification process (cf. p. I) the angle $\pi - \Phi$ represents the critical value of ω_{r_f}, at which failure of the composite takes place, whereas ω_{R_c} is the initial value of ω_{r_f}. Therefore, the establishment of an accurate relationship between the angles $\pi - \Phi$ and ω_{R_c} is absolutely necessary. In fact, such a relation follows immediately from eqns (13), p. I, and eqn (14), respectively, and reads

(15) $$\omega_{R_c} < \pi - \Phi, \quad \text{if} \quad \Delta\nu > (1+\nu_m)(1-2\nu_m)/3,$$

(16) $$\omega_{R_c} > \pi - \Phi, \quad \text{if} \quad \Delta\nu < (1+\nu_m)(1-2\nu_m)/3.$$

At this place some additional remarks should be made before proceeding with the further analysis. First of all, it can be stated that the general approach developed in the present study, reduces the entire problem of the elastic-plastic response of the considered fibrous composites to a plane plasticity problem, which has a close analogy to the well-known classical plane stress perfect plasticity problem (cf. for example Kachanov [6]). The latter problem also involves a yield ellipse, for which formally $\overset{*}{\varepsilon}{}_z^e \equiv 0$ and $\Phi = \pi/6$ hold true. By this analogy, the present problem can be approached in the following way. It is in fact a matter of routine procedures to prove that the yield ellipse from eqn (10), p. I, involves arcs of hyperbolicity and ellipticity as well as points of parabolicity. In particular, the arcs $\Phi < \omega < \pi - \Phi$ and $\pi - \Phi < \omega \leqq \pi$ of the latter ellipse are arcs of hyperbolicity and ellipticity, respectively, and the point $\omega = \pi - \Phi$ is a point of parabolicity. As the mathematical plasticity theory shows (Kachanov [6]), the regime of plastic redistribution of stresses, associated with these arcs and points, possess both overall and local specific features. Thus, one should necessarily distinguish between the latter regimes. This general conclusion reveals itself the importance of the above derived relations (14) — (16). They clearly indicate that different regimes of plastic deformation may develop in the matrix phase depending upon the value of the difference $\Delta\nu$ of Poisson's ratio. Thereby for large values of $\Delta\nu$ the relations (14) and (15) predict that a hyperbolic stress state will initially develop in the second plastic zone. In the case of small $\Delta\nu$-values (cf. relation (16)) there exists an elliptic state of stress in the plastic zone. Furthermore, the analysis from p. I proves that the process of matrix cooling induces a hyperbolic regime of plastic deformation in the matrix.

Thus in accordance with the foregoing considerations it would be reasonable to separate the analysis of the cases, corresponding to the relations (15) and (16) respectively. Thereby the terms "hyperbolic" and "elliptic" will be used in the following just in order to distinguish between these cases. The analysis itself will keep the structure of p. I. Certain general considerations, concerning the analogy with the plane stress perfect plasticity problem, are to be found in the authors' article Herrmann & Mihovsky [7].

HYPERBOLIC CASE

It is clear from the conclusions derived above that in this case eqn (8) is to be solved within the interval

(17) $$\omega_{R_c} \leqq \omega_{r_f} \leqq \pi - \Phi$$

along with the boundary condition

$$\varepsilon_z|_{\omega_{r_f}=\omega_{R_c}} = \overset{*e}{\varepsilon}_z, \tag{18}$$

where ω_{R_c} is defined by eqn (14) and satisfies relation (15), while the value $\overset{*e}{\varepsilon}_z$ follows from eqns (7), (13). In this case eqn (8) implies positive $d\omega_{r_f}$-values, i.e. the increasing loading, and thus the increasing axial strain ε_z leads to an increase in the angle ω_{r_f}. By applying the procedure from p. I the $\omega_{r_f}(\varepsilon_z)$-dependence can be obtained as an approximate solution of the problem, specified by eqns (8), (17), (18), respectively. This solution reads (when the principal terms are considered only)

$$\omega_{r_f} = \omega_{R_c} + b_1 \Lambda_1 \Delta\varepsilon_z, \tag{19}$$

where

$$b_1 = 2\nu_f \frac{\tan\Phi}{1-\nu_f}, \tag{20}$$

$$\Delta\varepsilon_z = \varepsilon_z - \overset{*e}{\varepsilon}_z. \tag{21}$$

The axial strain difference $\Delta\varepsilon_z^*$, at which failure of the unit cell takes place, follows formally from eqn (19) with $\omega_{r_f} = \pi - \Phi$ to be

$$\Delta\varepsilon_z^* = \frac{\pi - \Phi - \omega_{R_c}}{b_1 \Lambda_1}. \tag{22}$$

As it will be explained below, eqn (22) is a formal one. It assumes implicitly that the failure takes place before the entire plastification of the matrix, which is not the actual case in the considered problem (in contrast to the thermal one).

Equations (16), (17), (20) from p. I for the plastic zone radius R_c apply in the present case as well with the new ω_{R_c}-value from eqn (14). Therefore, by analogy with the thermal case and with eqn (19) the $R_c(\varepsilon_z)$-dependence reads

$$R_c^2(\Delta\varepsilon_z) = R_c^{*2}\left[1 - \left(1 - \frac{r_f^2}{R_c^{*2}}\right)\left(1 - \frac{\Delta\varepsilon_z}{\Delta\varepsilon_z^*}\right)^2\right]. \tag{23}$$

Finally, the condition of equilibrium of the axial forces

$$r_f^2 \sigma_z^f + (r_m^2 - R_c^2)\sigma_z^{me} + 2\int_{r_f}^{R_c} \sigma_z^{mp} r\,dr = r_m^2 \sigma_z^c \tag{24}$$

leads to the following forms of the $\sigma_z^c(\varepsilon_z)$-dependence again by the only consideration of principal terms

(25)
$$\Delta\sigma_z^c = \Delta\varepsilon_z E_m \left(1 + E_c - \frac{R_c^2}{r_m^2}\right),$$

or

(26)
$$\Delta\sigma_z^c = \Delta\varepsilon_z \frac{E_m(1+E_c)}{1 + \dfrac{2}{1+E_c}\dfrac{R_c^{*2} - r_f^2}{r_m^2}\dfrac{\Delta\varepsilon_z}{\Delta\varepsilon_z^*}}.$$

The function $\Delta\sigma_z^c(\Delta\varepsilon_z)$ is easily seen to be convex and to deviate smoothly from the straight $\sigma_z(\varepsilon_z)$-line defined by eqn (4). Further, the analysis of the composite response, eqn (26), in the sense of that from p. I, is now performable straightforwardly. But in the present case such a detailed analysis is actually not necessary for the following reason. Eqn (26) reflects the response of the composite within the short interval $[\overset{*e}{\varepsilon_z}, \varepsilon_z^{t.pl.}]$, where $\varepsilon_z^{t.pl.}$ stays for the value of the ε_z-strain at which total (complete) matrix plastification takes place.

Upon introducing

(27)
$$\Delta\varepsilon_z^{t.pl.} = \varepsilon_z^{t.pl.} - \overset{*e}{\varepsilon_z}$$

it is clear that eqns (22), (26) would be of actual importance if the failure of the composite takes place before the total plastification of the matrix, i.e. if $\Delta\varepsilon_z^* < \Delta\varepsilon_z^{t.pl.}$. Thus, the next specific question that needs to be cleared up is which of the two latter phenomena takes place at first. This question concerns, first of all, the determination of the $\varepsilon_z^{t.pl.}$-value.

In solving this question it should be firstly mentioned that in the present case the plastification of the matrix will lead to a reduction of the cross-sectional stresses in the remaining elastically deforming region. The plastic zone radius remains (in contrast to the thermal case) always much smaller than r_m, i.e. $R_c^* \ll r_m$, which is simply due to the smallness of the coefficient $(\pi - \Phi - \omega_{R_c})$ in the exponent in eqn (20), p. I. Accordingly, the themselves small σ_i^{me}-stresses ($i = r, \theta$), acting in the elastic matrix region $R_c \leq r \leq r_m$, decrease further. This result allows a consideration of the stress state in this region as approaching a state of pure axial tension with $\sigma_z^{me} = E_m\varepsilon_z$. Thus the plastification of this entire region takes suddenly place when $\sigma_z^{me} = \sigma_y$. This result defines the value $\varepsilon_z^{t.pl.}$ as

(28)
$$\varepsilon_z^{t.pl.} = \varepsilon_y = \frac{\sigma_y}{E_m}.$$

The foregoing equations allow to prove that the relation $\Delta\varepsilon_z^* > \Delta\varepsilon_z^{t.pl.}$ holds practically always true. This is consistent with the typical observations of the behaviour of the fibrous composites. Thereby the occurrence of the stage of elastic-plastic behaviour with a completely plastified matrix precedes the failure of the composite. Thus eqn (19) (respectively eqn (26)) is valid only in the interval $[\overset{*e}{\varepsilon}_z$, $\varepsilon_z^{t.pl.}]$. The quantity $\Delta\varepsilon_z^*$, defined by eqn (22), is itself not a real characteristic of the composite. Furthermore, the values of $\Delta\varepsilon_z^{t.pl.}$ and of the corresponding stress of total plastification $\sigma_z^{c,t.pl.}$ (respectively $\Delta\sigma_z^{c,t.pl.}$) follow from eqn (7), (13), (25), (27), (28) by considering principal terms only

$$\Delta\varepsilon_z^{t.pl.} = \frac{\sigma_y}{E_m}\frac{3(\Delta\nu)^2}{2(1+\nu_m)^2}, \tag{29}$$

$$\Delta\sigma_z^{c,t.pl.} = \sigma_z^{c,t.pl.} - \sigma_z^{c,pl.} = \sigma_y(1+E_c)\frac{3(\Delta\nu)^2}{2(1+\nu_m)^2}. \tag{30}$$

Upon introducing the notations

$$\Delta\varepsilon_{z,2} = \varepsilon_z - \varepsilon_y, \tag{31}$$

$$\Delta\sigma_{z,2}^c = \sigma_z^c - \sigma_z^{c,t.pl.}, \tag{32}$$

and by applying the equilibrium condition of the axial forces the $\sigma_z^c(\varepsilon_z)$-dependence for the considered stage can be given in the form

$$\Delta\sigma_{z,2}^c = E_m E_c \Delta\varepsilon_{z,2}. \tag{33}$$

Thereby eqn (33) is nothing else but the known lower bound estimation of the elastic-plastic response of the considered class of fibrous composites, derived by Hill [4]. The interpretation of eqn (33) in the "rule of mixture" sense with a negligible contribution of the plastified matrix phase (cf. Kelly [3] and the introduction to p. II) is straightforward.

A qualitative purely schematic illustration of the overall response of the considered composites, as predicted by the present analysis, is given in Fig. 1. Thereby the straight lines I and III correspond to eqns (4) and (33), respectively, while the straight line II is the linear approximation of the dotted one, to which eqn (26) corresponds. The strain $\varepsilon_z^* = \varepsilon_z^{t.pl.} + \Delta\varepsilon_{z,2}^*$ with $\varepsilon_z^{t.pl.}$ defined by eqn (28), is the strain at which the failure modes, predicted by the model of Herrmann & Mihovsky [1], start actually developing in the composite cell. How these failure modes occur upon the complete matrix plastification is a problem with the solution of which the analysis of the hyperbolic case will be entirely closed. To this regard eqn (8) proves that with further loading of the composite, i.e. upon $\sigma_z^{c,t.pl.}$, the angle ω_{r_f} further increases and finally approaches the angle $(\pi - \Phi)$. In addition, it can be shown that in accordance with the u_r-continuity condition at $r = R_c$, i.e. at the boundary between the two plastic zones, the angle ω_{R_c} increases as well but it remains, at

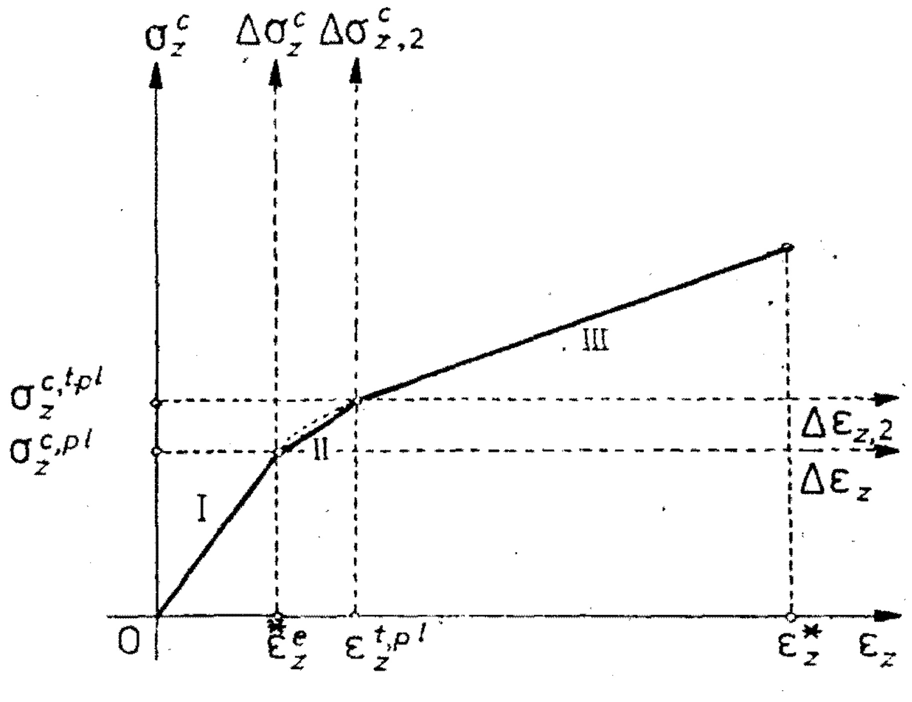

Fig. 1. Qualitative schematic illustration of the response of fibrous composites under longitudinal extension

the same time, smaller than ω_{r_f}. Thus, it is the latter angle that first achieves the critical value $(\pi-\Phi)$ to which the occurrence of the failure modes of the composites at the fibre-matrix interface corresponds.

It should be pointed out that these conclusions result, in fact, from a relatively complicated analysis. The latter involves a second yield ellipse (with $\overset{*}{\varepsilon}{}_z^e = \varepsilon_y$), the σ_r-continuity condition at $r = R_c$, as well as a jump in the σ_θ-stress at $r = R_c$. The occurrence of this jump results from the sudden change in the process of plastic stress redistribution, caused by the sudden entire matrix plastification. The stress state in the suddenly plastified matrix annulus $R_c \leqq r \leqq r_m$ with $\sigma_r \approx \sigma_\theta \approx 0$ corresponds to the vertex $\omega = \pi$ of the second yield ellipse. This vertex belongs to an arc of ellipticity of the latter. The necessity of introducing this ellipse reflects the fact that a sudden plastification of the elastically deformed matrix region corresponds to a value $\overset{*}{\varepsilon}{}_z^e$ of the axial strain in this region, which is equal to ε_y.

Further, eqn (17), p. I, with ω_{R_c} from eqn (14) now proves that with the behaviour of ω_{r_f} and ω_{R_c} described above the plastic zone radius R_c decreases with progressive loading. This is a natural result since the large plastic zone $R_c \leqq r \leqq r_m$ should be really expected to reduce the stress concentration within the thin plastic layer $r_f \leqq r \leqq R_c$, surrounding the fibre, and to reduce in this way its size R_c as well.

When solved in the terms of $\Delta\varepsilon_{z,2}$ and $\Delta\omega_{r_f,2} = \omega_{r_f} - \omega_{r_f}^{t.pl.}$, (cf. eqns (19), (29)) the u_r-continuity condition, eqn (8), will imply with $\omega_{r_f} = \pi - \Phi$ the actual critical $\Delta\varepsilon^*_{z,2}$-strain difference, respectively, the critical ε^*_z-strain (cf. Fig. 1) at which modes of failure of the composite will start developing. The determination of this critical strain difference $\Delta\varepsilon^*_{z,2}$ is a matter of simple computations, and the strain ε^*_z is equal to $\varepsilon^{t.pl.}_z + \Delta\varepsilon^*_{z,2}$ (cf. Fig. 1).

ELLIPTIC CASE

This case corresponds to relatively small values of $\Delta\nu$, for which the inequality (16) holds true. The stress concentration effect is smaller than in the foregoing case. The initial value of ω_{r_f}, i.e. ω_{R_c}, belongs to the interval $[\pi - \Phi, \pi]$. The speciality of this case is associated with the behaviour of the function $f_1(\omega_{r_f})$ from eqn (8). This function changes its sign when ω_{r_f} runs through the value

$$\bar{\omega}_{r_f} = \arctan[-(1 - 2\nu_f)\tan\Phi]. \tag{34}$$

The angle $\bar{\omega}_{r_f}$ obviously belongs to the interval $[\pi - \Phi, \pi]$. It is easy to prove at the same time that irrespectively of whether the ω_{R_c}-value is larger or smaller than $\bar{\omega}_{r_f}$, the change in the sign of the function $f_1(\omega_{r_f})$ at $\omega_{r_f} = \bar{\omega}_{r_f}$ guarantees that upon the occurence of the second plastic zone the angle ω_{r_f} in any case will achieve the value $\bar{\omega}_{r_f}$, and that further development of the matrix plastification process will be possible with this constant value $\bar{\omega}_{r_f}$ of ω_{r_f}. It is reasonable to accept that the progressive loading causes again an increase of the plastic zone radius R_c. Then, in accordance with eqn (17), p. I (with $\omega_{r_f} = \bar{\omega}_{r_f}$ now), this increase should be due to the increase of the angle ω_{R_c} in the interval $[\pi - \Phi, \pi]$. At the instant when $\omega_{R_c} = \pi$ the plastic zone radius R_c becomes infinitely large, i.e. sudden total matrix plastification takes place. Note, that the point $\omega = \pi$ belongs to an arc of ellipticity of the yield ellipse (p. I, eqn (11)).

The effect of constancy of the angle ω_{r_f} and of the sudden entire matrix plastification can be explained in the following way. As eqn (8) shows, in this case the angle ω_{r_f} changes upon the occurrence of the second plastic zone $r_f \leqq r \leqq R_c$ between the themselves close values ω_{R_c} and $\bar{\omega}_{r_f}$. The plastic zone radius R_c remains small again, i.e. comparable with the fibre radius r_f (p. I, eqn (17)). At the same time the thinness of the plastic zone reflects the very low stress concentration effect of the fibre. Since the plastification itself further reduces the latter effect, it can be assumed that at a certain instant of the plastification process the radial dependence of the stresses in the thin plastic zone becomes negligible. Then, due to the σ_r-continuity condition at $r = r_f$, the fibre and the thin plastic coating around it could be considered at this instant just as forming an "elastic" core $r_f \leqq r \leqq R_c$ of the composite cell, which expands with an "increasing" Poisson's ratio ν_c. The core is "elastic" in the sense that the existing radial stress in it satisfies, as in the homogeneous linearly elastic fibre material, the relation $\sigma_r = \sigma_r|_{r=R_c}$ (p. I, eqns (1)). The "increase" of the ν_c-ratio is due to the plastic incompressibility of the strain in the thin plastic coating as well as due to the expansion of the latter, i.e. due to the increase of its volume fraction. With the concept of the core formation the u_r-continuity condition at $r = r_f$, i.e. within the core now, may be considered as identically satisfied irrespectively of the values of the angle ω_{r_f}. The latter keeps actually the value $\overline{\omega}_{r_f}$. The core spreads into the matrix phase with the increasing ν_c-value and thus reduces the stresses σ_i^{me}, $i = r, \theta$, in the remaining elastic region, since the latter are proportional to the itself decreasing difference $\nu_m - \nu_c$, cf. eqns (1), p. I and eqn (1), respectively. Obviously, this is the above considered stage of deformation with increasing ω_{R_c}-values ($\omega_{R_c} \longrightarrow \pi$). Thus the instant of complete matrix plastification $\omega_{R_c} = \pi$ corresponds to that one at which ν_c becomes equal to ν_m (cf. eqn (14) with $\Delta\nu = \nu_m - \nu_c$).

It should be mentioned without discussing the details that the analysis of the unit cell behaviour upon the instant of complete matrix plastification follows the same basic lines as in the foregoing case. It predicts, as it should be expected, the same response (cf. eqn (33)). The distinguishing feature between the two cases concerns in fact the length of the transitional interval $[\overset{*}{\varepsilon}{}_z^{e}, \varepsilon_z^{t.pl.}]$ (cf. Fig. 1). This interval proves to be even shorter in the present case than the itself short interval from the hyperbolic case.

The basic features of the development of the plastic deformation process, as well as of its influence on the overall response of a fibrous composite, have been considered above in sufficient detail and do not need to be additionally analyzed in a separate section as in p. I. Nevertheless, it would be of interest to summarize both the common and the specific features of the thermal and the mechanical response of the considered class of composites from the view-point of the general approach developed in this study. Such a summary is presented in the next section along with a brief consideraton of these features, concerning the possible applications of the general approach to some problems of the practice of the fibrous composites.

CONCLUDING REMARKS

The general approach, developed in the present study, predicts a realistic elastic-plastic response of the considered class of fibrous composites under both thermal and mechanical loading condition. Quantitatively, the predicted response reflects both the geometrical and the mechanical properties of the composite structure. It is consistent with the "rule of mixtures" description of the composites behaviour commonly adopted in the engineering practice. The response itself is derived as an overall quantitative estimation of the characteristics of the processes of elastic and plastic deformation, respectively, developing simultaneously within the composite structure.

In accordance with the analysis of both the overall and these specific features of these processes different regimes in the development of the matrix plastification process may occur, depending upon the loading status or/and the properties of the constituents. The response of a fibrous composite under the condition of matrix cooling corresponds to a regime of monotonous increase of the plastic zone size. Similar regimes develop initially under longitudinal extension as well. The latter cover, as a rule, a short interval of axial strain changes and are followed by the phenomenon of the sudden entire matrix plastification. The general approach allows to draw a clear analogy between these regimes and the regimes developing in the classical plane stress perfect plasticity problem. From the point of view of this analogy the phenomena of progressive increase of the plastic zone size and entire sudden matrix plastification just reflect the essential properties of the corresponding set of governing equations, if the latter are of the hyperbolic or elliptic type, respectively. Further, the approach relates the change of the type of this set of equations to a parabolic one with the occurrence of specific failure phenomena, connected with the considered class fibrous composites.

The approach described above accounts in a special way for the mutually conquering effects of the matrix ductility and the fibre stiffness. The quantity $\overset{*}{\varepsilon}{}_z^e$ involved in this approach proves to be a reliable average measure of the interactions between these effects. Its identification in the thermal problem is of importance. To this regard corresponding practical procedures are proposed.

As it was mentioned in the introduction to p. I, the basic effects of the matrix plasticity concern its influence on the overall composite response and the improvement of the fracture resistance of the composites to existing structural defects. Along with the clarification of the first of these effects the present approach allows to derive definite conclusions with respect to the second one as well. Thereby it is clear to this regard that in the case of longitudinal extension the plasticity of the matrix reduces the cross-sectional stresses. Accordingly, if defects are present in the matrix phase, which are sensitive to these stresses, then one should account for the possible growth of such defects only within the linear elastic stage of the composite behaviour. The plasticity of the matrix really improves the resistance of the composites to such defects. Typical defects of this type are, for example, the relatively short cracks which, when reffered to the unit cell cross-section, may be considered as radial cracks. Such cracks occur very often during the processes of thermal treatment involved in the fabrication of the composites.

The plasticity of the matrix does not reduce the sensitivity of the composites to such cracks under the conditions of matrix cooling. In this case the circumferential stress at the front $r = R_c$ of the plastic zone is relatively large. The enlargement of this zone results in a relative increase of this stress in the points, traversed by the front. Therefore, if a radial crack exists in the elastic matrix region then with progressive matrix cooling the elastic-plastic boundary will approach the crack tip and imply larger stress concentration there. Such a crack, even if it was in equilibrium in the elastic stage of the composite behaviour, may start propagating due to the progressive process of matrix plastification. Thus, in that case the plasticity of the matrix does *not* improve the fracture resistance of a composite. Moreover, the same relatively large circumferential stress, carried by the propagating plastic zone front, may be considered as the reason for the occurrence of such cracks during the processes of thermal treatment involved in the fabrication of the composites.

Thereby, as it was mentioned, the problem of matrix cooling is considered as modelling the real fabrication problem of cooling the entire composite structure. In fact, the basic lines of the analysis from p. I apply to the latter problem as well if, roughly speaking, the term α_m in the thermal analysis is replaced by $\Delta\alpha = \alpha_m - \alpha_f$.

One may then expect by analogy with the case of longitudinal extension that depending upon the specific value of $\Delta\alpha$ different regimes of plastic deformation may develop in the matrix phase during the fabrication process of cooling of the entire composite structure. Accordingly, by using the present approach a development of fabrication technologies should be possible which would at least reduce, if not entirely prevent, the undesired radial cracking of the composites and therefore also the propagation of existing radial cracks respectively. Similar applications of the approach, based upon the suitable choice of the $(\Delta\alpha, \Delta\nu)$-combinations, may be of importance in problems concerning both the load-bearing capacities and the crack sensitivity of the considered composites at low temperatures.

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