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OPTIMIZATION AND RELIABILITY OF AN ELECTRICAL NETWORK

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Цветана Недева. ОПТИМИЗАЦИЯ И НАДЕЖДНОСТЬ ЕЛЕКТРОЕНЕРГИЙНОЙ СИСТЕМЫ

Рассматриваются электроэнергийная система и тесты ее надеждности. Чтобы получить надеждную систему надо поменить (дополнить) некоторые из ее ветвей. Задача об оптимальном дополнении системы сформулирована как задача двухэтапного стохастического программирования. Эту задачу можно решать стохастическими квазыградиентными методами.

Cvetana Nedeva. OPTIMIZATION AND RELIABILITY OF AN ELECTRICAL NETWORK

An electric power system and tests for its reliability are considered. In order to get a reliable network, we have to change some of the branches. The problem for the optimal exchange is formulated as a two-stage stochastic programming problem, solvable by stochastic quasigradient methods.

The configuration of an electrical network is represented by a graph G with n nodes, M being the set of the branches. For every node p a number P_p — the power of the node p — is given, $p = 1, 2, \ldots, n$. Every branch $(p,q) \in M$ is characterized by its conductivity b_{pq}. Then the phases of the tensions of the nodes are the solution of the linear system of the power flow

$$\sum_{q=1}^{n} b_{pq}(\theta_p - \theta_q) = P_p, \quad p = 1, 2, \ldots, n. \tag{1}$$

The solution is technically feasible when it possesses the conditions

$$(2) \qquad |\theta_p - \theta_q| \leq \tau_{pq}, \quad (p,q) \in M.$$

A probability model for estimating the reliability of the electrical network is given in [1].

Here is considered a number of tests $(\pi_0, \pi_1, \dots, \pi_k)$ which correspond to different states of the network: π_0 — perfect working order, π_i — the branch (p_i, q_i) vanishes ($b_{p_i q_i} = 0$), $i = 1, 2, \dots, k$. Thus it is possible the solution of the corresponding system (1) to be technically unfeasible for some state π_i and this means that the electrical network is not reliable. In order to get a realible system, one has to change some of the branches, which corresponds to the increasing of b_{pq} for some (p,q). Such kind of changes are necessary when a developing of an electrical system is planned. A problem of computer aided design of a network when loads (P_p) may vary is discussed in [2].

Let us consider the following optimal reliability problem: Find $x = (x_{pq}) \in X^0$ — changes of the branch conductivities b_{pq} — such that to minimize the objective function (the price of new branches)

$$(3) \qquad f(x) = \sum_{(p,q)\in M} w_{pq} x_{pq},$$

subject to

$$(4) \qquad \sum_{q=1}^{n} b_{pq}(x,\pi)(\theta_p - \theta_q) = P_p, \quad p = 1, 2, \dots, n,$$

$$(5) \qquad |\theta_p - \theta_q| \leq \tau_{pq}, \quad (p,q) \in M(x,\pi).$$

The conditions (4) and (5) have to be fulfilled for every $\pi = \pi_i$, $i = 1, 2, \dots, k$. Here by $b_{pq}(x,\pi)$ the conductivity of the branch (p,q) is denoted (it depends eventually on the solution x and on the state π), $M(x,\pi)$ is the set of branches, X^0 is the set of the feasible changes from the technical point of view, and w_{pq} is a price.

The problem formulated is a linear programming problem with very large number of variables. The number of the variables x_{pq} is relatively small, but the vector $\theta(x,\pi) = (\theta_1(x,\pi), \dots, \theta_n(x,\pi))$ is obtained for every particular combination of x and π. So the number of the unknowns becomes very great. The structure of the system (4) is a block one and this fact is used in [3, 4] for solving similar problems.

An alternative way is to formulate the problem (3)–(5) as a two-stage stochastic programming problem. Let us assume that the state π of the electrical network is a random variable with known distribution function. The idea for the implementation of two-stage stochastic programming is based on the following:

— the variables x_{pq} are independent on the state π and have to be determined before the observation on π;

— the variables θ_p are determined after the moment when the values of x_{pq} are chosen and the state π is known;

— the estimation and the conclusion for the goodness of the choice of the solution x are based on the values of θ_p for all possible but concrete $\pi = \pi_0, \pi_1, \dots, \pi_k$.

The application of the two-stage stochastic programming is rather natural here: there are two levels of the solution procedure (the first level is the determination of x and the second one is the determination of θ for given x and π) — stachastic and optimization. Some difficulties arise because of the estimation mentioned above: we need a criterion which optimal value will show whether x is a proper solution of the problem or it is not.

For fixed x and π such kind of objective function is

$$F^0(x,\pi) = \min_{\theta}\left\{ f^0(x,\theta,\pi) = \max_{p}\left|\sum_{q=1}^{n} b_{pq}(x,\pi)(\theta_p - \theta_q) - P_p\right|\right\},$$

subject to

$$|\theta_p - \theta_q| \le \tau_{pq}, \quad (p,q) \in M(x,\pi),$$

and for every π — its mean value

$$F(x) = E(F^0(x,\pi)).$$

Obviously, if $F(x) = 0$, then the choice of x is good; otherwise, there exists some π_i such that the system of conditions (4), (5) is unsolvable.

Taking into account the criterion (3), we form the function

$$G(x) = f(x) + L.F(x),$$

where L is a large positive number, for example $L \ge \sum w_{pq}$.

So we obtain the following two-stage stochastic programming problem: Minimize the function

$$G(x) = f(x) + L.E\{F^0(x,\pi) = f^0(x,\theta(x,\pi),\pi)\},$$

where $\theta(x,\pi)$ is a solution of the "inner" problem

$$\min\left\{ f^0(x,\theta,\pi) = \max\left|\sum_{q=1}^{n} b_{pq}(x,\pi)(\theta_p - \theta_q) - P_p\right|\right\},$$

subject to

$$|\theta_p - \theta_q| \le \tau_{pq}, \quad (p,q) \in M(x,\pi).$$

Let us note that the exact values of the function $G(x)$ can not be computed and $G(x)$ is non-differentiable. Nevertheless, the optimization problem is numerically solvable by stochastic quasigradient methods [5].

Another feature of this approach is that it is useful in computer aided design of new electrical systems. Similar problems arise often when a solution under uncertainly is taken for a system which topology is characterized by a graph.

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