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ABOUT THE FIRST CROSSING OF THE POISSON PROCESS WITH A CURVED UPPER BOUNDARY

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The paper is concerned with the distribution of the first crossing of a simple Poisson process trajectory with an upper boundary. Exact formula is derived when the upper boundary has a vertical asymptote.

Keywords: risk theory, Poisson process, first crossing time, upper boundary

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1. INTRODUCTION

Many problems in risk, queuing and storage theories can be reduced to the study of the first crossing time or level of a given boundary with a trajectory of a certain stochastic process. Such problems have been mainly investigated for continuous time Gaussian or similar to Gaussian processes. In case of non-linear boundaries and of discrete-state space, the literature is rather sparse, and it treats only the ordinary or compound Poisson process. The reader is referred to Lundberg (1903), Cramér (1955), Whittle (1961), Daniels (1963), Gallot (1966, 1993), Zacks (1991), Stadje (1994), Schäl (1993), Picard and Lefèvre (1997), Kalashnikov (1996).

In the present work, the interest will be focused on the classical continuous time model of an insurance company, i.e. the Poisson model.

Suppose that $\xi_1, \xi_2, \xi_3, \ldots$ are independent and exponentially distributed with

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parameter λ, so that

$$k(t) = \max\{n : \xi_1 + \xi_2 + \cdots + \xi_n \le t\} \tag{1}$$

defines an ordinary Poisson process $k(t)$, $t \ge 0$. We shall interpret

$$S_n := \xi_1 + \xi_2 + \cdots + \xi_n \tag{2}$$

as the moment of the n-th insurance claim. If η_n represents the amount of the n-th claim, then

$$Z_t := \sum_{i \le k(t)} \eta_i \tag{3}$$

represents the total amount of claims to time t. The stochastic process Z_t, $t \ge 0$, coincides with $k(t)$, $t \ge 0$, when $\eta_i \equiv 1$, $i = 1, 2, \ldots$

Let

$$U_t = f(t) - Z_t, \tag{4}$$

where $f(t)$ is a non-decreasing real function defined on the set $\mathbb{R}_+ = \{x : x \ge 0\}$. In the classical risk model, usually the function $f(t)$ has the form

$$f(t) = u + ct,$$

where c is the premium income per unit time, and $u := f(0)$ is the initial surplus.

Define the ruin time T as

$$T := \inf\{t : U_t \le 0,\ t > 0\}, \tag{5}$$

i.e. T is the time of the first crossing of the trajectory $t \to Z_t$ with the boundary $t \to f(t)$ (disregarding the origin when $f(0) = 0$).

Recently, Picard and Lefèvre (1997) have investigated the compound Poisson risk model when the integer valued random variables $\eta_1, \eta_2, \ldots$ are independent identically distributed and the sequences $\xi_1, \xi_2, \ldots$ and $\eta_1, \eta_2, \ldots$ are independent. They derived the expressing for the ruin probability $\mathrm{P}(T \le x)$ in terms of generalized Appell's polynomials under the assumption

$$\mathrm{P}(\eta_i \ge 1) = 1 \quad \text{and} \quad \lim_{t \to \infty} f(t) = +\infty.$$

Our purpose is to find the ruin probability $\mathrm{P}(T \le x)$ in the particular case when $f(t)$ has a vertical asymptote (i.e. $\lim_{t \uparrow v} f(t) = \infty$ for some $v > 0$) and $\eta_i \equiv 1$.

2. THE PROBABILITY OF RUIN IN FINITE TIME

It is worth noting that the distribution of T is defective ($\mathrm{P}(T = \infty) > 0$). Further we shall use the quantities

$$v_n : f^{-1}(n), \quad n = 0, 1, 2, \ldots, \tag{6}$$

where the inverse function $f^{-1}(x)$ is defined by

$$f^{-1}(x) := \inf\{y : f(y) \ge x\}.$$

Obviously, $v_0 = 0 \leq v_1 \leq v_2 \leq \ldots$ and

$$\lim_{n\to\infty} v_n = v. \tag{7}$$

We shall use the formula for the non-ruin probability $\mathrm{P}(T > x)$ derived by Ignatov and Kaishev (1997) in the form

$$\mathrm{P}(T > x) = \sum_{n\geq 0} e^{-x}\left[\sum_{i=0}^{n}(-1)^i\delta(v_1,\ldots,v_i)\sum_{j=0}^{n-i}\frac{x^j}{j!}\right].I_{[v_n,v_{n+1})}(x), \tag{8}$$

where $\delta(v_1,\ldots,v_i) = 1$ for $i = 0$ and

$$\delta(v_1,\ldots,v_i) = \det\begin{pmatrix} v_1 & 1 & 0 & \ldots & 0 & 0\\ \frac{v_2^2}{2!} & v_2 & 1 & \ldots & 0 & 0\\ \frac{v_3^3}{3!} & \frac{v_3^2}{2!} & v_3 & \ldots & 0 & 0\\ \ldots & \ldots & \ldots & \ldots & & \\ \frac{v_{i-1}^{i-1}}{(i-1)!} & \frac{v_{i-1}^{i-2}}{(i-2)!} & \frac{v_{i-1}^{i-3}}{(i-3)!} & \ldots & v_{i-1} & 1\\ \frac{v_i^i}{i!} & \frac{v_i^{i-1}}{(i-1)!} & \frac{v_i^{i-2}}{(i-2)!} & \ldots & \frac{v_i^2}{2!} & v_i \end{pmatrix} \tag{9}$$

for $i = 2, 3, \ldots$, and $I_{[v_n,v_{n+1})}(x)$ is the indicator function of the interval $[v_n, v_{n+1})$.

The formula (8) is obtained under the assumptions $1 \equiv \eta_1 \equiv \eta_2 \equiv \cdots$ and the parameter $\lambda \equiv 1$ of the sequence $\xi_1, \xi_2, \ldots$ We shall assume now that the last assumptions are fulfilled.

The main result in this section is the following

Theorem. *If $f(t)$ is such that $v_n \uparrow v$, then*

$$\mathrm{P}(T > x) = \begin{cases} \sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i), & x \geq v,\\ \sum_{n\geq 0} e^{-x}\left[\sum_{i=0}^{n}(-1)^i\delta(v_1,\ldots,v_i)\sum_{j=0}^{n-i}\frac{x^j}{j!}\right].I_{[v_n,v_{n+1})}(x), & 0 \leq x < v. \end{cases} \tag{10}$$

Proof. To prove the theorem, we shall use the next two lemmas.

Lemma 1. *For the determinants $\delta(v_1,\ldots,v_n)$ we have the identities*

$$\delta(cv_1,\ldots,cv_n) = c^n\delta(v_1,\ldots,v_n), \tag{11}$$

$$\delta(v_1 + c,\ldots,v_n + c) = \delta(v_1,\ldots,v_n) - \delta(v_1,\ldots,v_{n-1},-c), \tag{12}$$

$$\delta(v_1,\ldots,v_n) = (-1)^{n-1}\delta(v_n - v_1,\ldots,v_n - v_{n-1},v_n). \tag{13}$$

Proof. The identity (11) follows immediately from the definition of a determinant as a sum of certain products, in our case of the form $(-1)^h v_1^{j_1} v_2^{j_2} \dots v_n^{j_n}$, where h is a suitable integer and $j_1 + j_2 + \dots + j_n = n$.

Let us introduce the matrix

$$\Delta(v_1+c,\dots,v_n+c) := \begin{pmatrix} \frac{(v_1+c)^1}{1!} & 1 & 0 & \dots & 0 \\ \frac{(v_2+c)^2}{2!} & \frac{(v_2+c)^1}{1!} & 1 & \dots & 0 \\ \frac{(v_3+c)^3}{3!} & \frac{(v_3+c)^2}{2!} & \frac{(v_3+c)^1}{1!} & \dots & 0 \\ \dots & \dots & \dots & \dots & \\ \frac{(v_n+c)^n}{n!} & \frac{(v_n+c)^{n-1}}{(n-1)!} & \frac{(v_n+c)^{n-2}}{(n-2)!} & \dots & \frac{(v_n+c)^1}{1!} \end{pmatrix},$$

then $\det(\Delta(v_1+c,\dots,v_n+c)) = \delta(v_1+c,\dots,v_n+c)$.

If we add elements of the $(j+1)$-th column multiplied by $\frac{(-c)^j}{j!}$ to the elements of the first column for $j = 1, \dots, n-1$, we get

$$\delta(v_1+c,\dots,v_n+c) = \det \begin{pmatrix} \frac{v_1^1}{1!} & 1 & \dots & 0 \\ \frac{v_2^2}{2!} & \frac{v_2+c}{1!} & \dots & 0 \\ \dots & \dots & \dots & \\ \frac{v_{n-1}^{n-1}}{(n-1)!} & \frac{(v_{n-1}+c)^{n-2}}{(n-2)!} & \dots & 1 \\ \frac{v_n^n}{n!} - \frac{(-c)^n}{n!} & \frac{(v_n+c)^1}{(n-1)!} & \dots & \frac{(v_n+c)^1}{1!} \end{pmatrix}. \quad (14)$$

Indeed, for the element in the first column and the i-th row we have

$$\frac{(v_i+c)^i}{i!} + \frac{(-c)^1(v_i+c)^{i-1}}{1!(i-1)!} + \frac{(-c)^2(v_i+c)^{i-2}}{2!(i-2)!} + \dots + \frac{(-c)^i(v_i+c)^0}{i!0!}$$

$$\equiv \frac{1}{i!}\left(\frac{i!}{0!i!}(v_i+c)^0(v_i+c)^i + \frac{i!}{1!(i-1)!}(-c)^1(v_i+c)^{i-1}\right.$$

$$\left. + \frac{i!}{2!(i-2)!}(-c)^2(v_i+c)^{i-2} + \dots + \frac{i!}{i!0!}(-c)^i(v_i+c)^0\right)$$

$$\equiv \frac{1}{i!}(-c+v_i+c)^i = \frac{v_i^i}{i!} \quad (15)$$

for $i = 1, \dots, n-1$.

For $i = n$ it is easy to find the identity

$$\frac{(v_n+c)^n}{n!}+\frac{(-c)^1(v_n+c)^{n-1}}{1!(n-1)!}+\frac{(-c)^2(v_n+c)^{n-2}}{2!(n-2)!}+\cdots+\frac{(-c)^{n-1}(v_n+c)^1}{(n-1)!1!}$$
$$\equiv \frac{v_n^n}{n!}-\frac{(-c)^n}{n!}.$$

A similar construction will be used to the elements of the second column and so on.

Finally, we get

$$\delta(v_1+c,\ldots,v_n+c)$$
$$=\det\begin{pmatrix} \frac{v_1^1}{1!} & 1 & \ldots & 0 \\ \frac{v_2^2}{2!} & \frac{v_2^1}{1!} & \ldots & 0 \\ \cdots & \cdots & \cdots & \\ \frac{v_{n-1}^{n-1}}{(n-1)!} & \frac{v_{n-1}^{n-2}}{(n-2)!} & \ldots & 1 \\ \frac{v_n^n}{n!}-\frac{(-c)^n}{n!} & \frac{v_n^{n-1}}{(n-1)!}-\frac{(-c)^{n-1}}{(n-1)!} & \ldots & \frac{v_n^1}{1!}-\frac{(-c)^1}{1!} \end{pmatrix}. \tag{16}$$

From the well-known property of determinants and the form of the elements of the last row in (16) we obtain the identity (12).

The identity (13) follows from (11) and (12). Indeed, using (12) with $c=v_n$, we have

$$\delta(v_n-v_1,\ldots,v_n-v_{n-1},v_n+0)=\delta(-v_1,\ldots,-v_{n-1},0)-\delta(-v_1,\ldots,-v_{n-1},-v_n)$$
$$=-\delta(-v_1,\ldots,-v_{n-1},-v_n)=(-1)^{n+1}\delta(v_1,\ldots,v_n).$$

In the second equality we use the fact that $\delta(-v_1,\ldots,-v_{n-1},0)\equiv 0$. In the third equality we have used (11).

The proof of Lemma 1 is complete.

Lemma 2. *If* $v_n\uparrow v$, *then the series* $\sum_{n=0}^{\infty}|\delta(v_1,\ldots,v_n)|$ *are convergent, i.e.*

$$\sum_{n=0}^{\infty}|\delta(v_1,\ldots,v_n)|<+\infty. \tag{17}$$

Proof. Let s be chosen such that

$$0\le v-v_{s+n}\le\frac{1}{3} \tag{18}$$

for each $n\ge 0$. We shall use the Laplace's expansion of a determinant and for this purpose we introduce the notation $\delta_{j_1,\ldots,j_s}^{r_1,\ldots,r_s}(v_1,\ldots,v_s)$ for the determinant formed from $\delta(v_1,\ldots,v_s)$ by using the elements in the rows r_1, …, r_s and the columns

$j_1, \ldots, j_s$. Let us expand the determinant $\delta(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$ applying the Laplace's formula

$$\delta(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$= \sum_{(j_1,\ldots,j_s)\in C_s^{s+n}} (-1)^h \delta^{1,\ldots,s}_{j_1,\ldots,j_s}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$\times \delta^{s+1,\ldots,s+n}_{j_{s+1},\ldots,j_{s+n}}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}), \quad (19)$$

where the summation is taken over the set C_s^{n+s}, the set of $\binom{n+s}{s}$ subsets $(j_1, \ldots, j_s)$ from the integers $(1, 2, \ldots, s+n)$. It is easy to find that

$$\delta^{1,2,\ldots,s}_{j_1,\ldots,j_s}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) = 0$$

when at least one of the indices $j_1, \ldots, j_s$ is greater than $s+1$. Therefore in this case we obtain

$$\delta(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$= \sum_{(j_1,\ldots,j_s)\in C_s^{s+1}} (-1)^h \delta^{1,\ldots,s}_{j_1,\ldots,j_s}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$\times \delta^{s+1,\ldots,s+n}_{j_{s+1},\ldots,j_{s+n}}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}). \quad (20)$$

For the sake of simplicity we shall denote the subset $(j_1, \ldots, j_s) \in C_s^{s+1}$ by $(1, 2, \ldots, i-1, \hat{i}, i+1, \ldots, s+1)$ if $j_k \neq i$ for $k = 1, \ldots, s$, and also we shall choose $j_{s+1} = i$, $j_{s+2} = s+2, \ldots, j_{s+n} = s+n$.

Now we can rewrite (20) as

$$\delta(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$= \sum_{i=1}^{s+1} (-1)^h \delta^{1,2,\ldots,s}_{1,\ldots,i-1,\hat{i},i+1,\ldots,s+1}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$$

$$\times \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}). \quad (21)$$

To prove the inequality (17), it is enough to prove that

$$\sum_{n=0}^{\infty} |\delta(v_1, \ldots, v_{s+n})| < \infty \quad (22)$$

for some positive integer s.

From (19) and (21) we get

$$\sum_{n=0}^{\infty} |\delta(v_1, \ldots, v_{s+n})| = \sum_{n=0}^{\infty} |\delta(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})|$$

$$\leq \sum_{n=0}^{\infty} \sum_{i=1}^{s+1} \left| \delta^{1,2,\ldots,s}_{1,\ldots,i-1,\hat{i},i+1,\ldots,s+1} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right|$$

$$\times \left| \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right|$$

$$= \sum_{i=1}^{s+1} \sum_{n=0}^{\infty} \left| \delta^{1,2,\ldots,s}_{1,\ldots,i-1,\hat{i},i+1,\ldots,s+1} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right|$$

$$\times \left| \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right| . \tag{23}$$

Let us recall the Hadamard's inequality for a determinant of order n and value D with real or complex elements a_{ij}:

$$|D|^2 \leq \prod_{i=1}^{n} \left(\sum_{j=1}^{n} |a_{ij}|^2 \right) , \tag{24}$$

and obviously,

$$|D| \leq \prod_{i=1}^{n} \left(\sum_{j=1}^{n} |a_{ij}| \right) . \tag{25}$$

From (25) and $v_{s+n} - v_1 \leq v, \ldots, v_{s+n} - v_{s+n-1} \leq v$, $v_{s+n} \leq v$ for the determinant $\delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$ we have

$$\left| \delta^{1,2,\ldots,s}_{1,\ldots,i-1,\hat{i},i+1,\ldots,s+1} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right| \leq e^{sv} \tag{26}$$

for $i = 1, \ldots, s+1$.

For the determinant $\left| \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right|$ it is possible to prove that for each constant c

$$\delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (c(v_{s+n} - v_1), \ldots, c(v_{s+n} - v_{s+n-1}), cv_{s+n})$$

$$= c^{n+s-i+1} \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}). \tag{27}$$

The determinant $\delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n}(v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n})$ depends only on $v_{s+n} - v_{s+1}, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}$. Consequently, using inequalities (21), we have

$$v_{s+n} - v_{s+i} \leq v - v_{s+i} \leq \frac{1}{3} . \tag{28}$$

From (18), (27) and (28) we obtain

$$\left| \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (v_{s+n} - v_1, \ldots, v_{s+n} - v_{s+n-1}, v_{s+n}) \right|$$

$$= \left(\frac{1}{3} \right)^{n+s-i+1} \delta^{s+1,\ldots,s+n}_{i,s+2,\ldots,s+n} (3(v_{s+n} - v_1), \ldots, 3(v_{s+n} - v_{s+n-1}), 3v_{s+n})$$

$$< \left(\frac{1}{3} \right)^{n+s-i+1} e^{n-i} e^{3v} . \tag{29}$$

Replacing the determinants in (23) with the upper bounds in (26) and (29), we get

$$\sum_{i=1}^{s+1}\sum_{n=0}^{\infty} e^{sv}\left(\frac{1}{3}\right)^{n+s-i+1} e^{n-1}e^{3v} = e^{(s+3)v}\sum_{i=1}^{s+1}\left(\frac{1}{3}\right)^{s-i+2}\sum_{n=0}^{\infty}\left(\frac{e}{3}\right)^{n-1}$$

$$= e^{(s+3)v}\left(\frac{1}{3}\right)^{s+2}\sum_{i=1}^{s+1}\left(\frac{1}{3}\right)^{-i}\left(\frac{3}{e}\right)\frac{1}{1-\frac{e}{3}} < \infty.$$

The proof of Lemma 2 is complete.

Proof of the theorem. The expression for the probability $\mathrm{P}(T > x)$ when $0 \le x < v$ follows immediately from formula (8). Indeed, the probability $\mathrm{P}(T > x)$ depends only on the form of the upper boundary $f(t)$ in the interval $[0, x)$, therefore we can imagine that the condition $\lim_{t\to\infty} f(t) = \infty$ is true and in this case we can use the formula (8). It is clear that

$$\mathrm{P}(T \ge x) = \mathrm{P}(T \ge v) = \lim_{y\uparrow v}\mathrm{P}(T \ge y) \quad \text{for } x \ge v.$$

Now we can see that $\lim_{y\uparrow v}\mathrm{P}(T \ge x) = \sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i)$. The formula (8) may be expressed as

$$\mathrm{P}(T \ge x) = \sum_{n=0}^{\infty}\left(\sum_{i=0}^{n}(-1)^i\delta(v_1,\ldots,v_i)e^{-x}\sum_{j=0}^{n-i}\frac{x^j}{j!}\right).I_{[v_n,v_{n+1})}(x)$$

$$= \sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i).g_i(x), \tag{30}$$

where $g_0(x) = 1$ for $x \in [0, v)$ and for $i \ge 1$

$$g_i(x) = \begin{cases} 0, & x \in [0, v_n), \\ e^{-x}\sum_{j=0}^{n-i}\frac{x^j}{j!}, & x \in [v_n, v_{n+1}). \end{cases}$$

When $x \to v$, we have $n \to \infty$, so we get

$$\lim_{x\uparrow v} g_i(x) = 1, \quad i = 0, 1, \ldots \tag{31}$$

Since $|g_i(x)| \le 1$, $x \in [0, v)$, we have

$$\sum_{i=0}^{\infty}|(-1)^i\delta(v_1,\ldots,v_i)g_i(x)| \le \sum_{i=0}^{\infty}|\delta(v_1,\ldots,v_i)|. \tag{32}$$

Combining (32) and Lemma 2, we get that the series $\sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i)g_i(x)$ is uniformly convergent. Consequently, taking into account (31), we have

$$\lim_{x\uparrow v}\sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i)g_i(x)=\sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i)\lim_{x\uparrow v}g_i(x)$$
$$=\sum_{i=0}^{\infty}(-1)^i\delta(v_1,\ldots,v_i).$$

The proof of the theorem is complete.

REFERENCES

1. Lundberg, F. I. Approximerad fromställning af sannolifhetstunktionen, II. Återförsäkring av kollektivisker. Almquist & Wiksell, Uppsala, 1903.
2. Cramér, H. Collective risk theory. Jubilee volume of Försäkringsaktieboleget Skandia, Stockholm, 1955.
3. Whittle, P. Some exact results for one-sided distribution tests of the Kolmogorov-Smirnov type. *Ann. Math. Statist.*, **32**, 1961, 499–505.
4. Daniels, H. E. The Poisson process with a curved absorbing boundary. *Bull. Inter. Statist. Inst.*, 34th session, **40**, 1963, 994–1008.
5. Gallot, S. F. L. Asymptotic absorption probabilities for a Poisson process. *J. Appl. Prob.*, **3**, 1966, 445–452.
6. Gallot, S. F. L. Absorption and first passage times for a compound Poisson process in a general upper boundary. *J. Appl. Prob.*, **30**, 1993, 835–850.
7. Thorin, O. Probabilities of ruin. *Scand. Actuarial J.*, 1982, 65–102.
8. Zacks, S. Distributions of stopping times for Poisson process with linear boundaries. *Commum. Statist. Stoch. Models*, **7**, 1991, 233–242.
9. Stadje, W. Distribution of first-exit times for empirical counting and Poisson processes with moving boundaries. *Commun. Statist. Stoch. Models*, **9**, 1991, 91–103.
10. Schäl, M. On hitting times for jump-diffusion process with past depend local characteristics. *Stoch. Proc. Appl.*, **47**, 1993, 131–142.
11. Picard, Ph., C. Lefèvre. The Probability of Ruin in Finite Time with Discrete Claim Size Distribution. *Scand. Actuarial J.*, **1**, 1997, 58–69.
12. Picard, Ph., C. Lefèvre. First Crossing of Basic Counting Processes with Lower Non-Linear Boundaries. A Unified Approach through Pseudo-Polynomials (I). *Adv. Appl. Prob.*, **28**, 1996, 853–876.
13. Kalashnikov, V. V. Two-Sided Bounds of Ruin Probabilities. *Scand. Actuarial J.*, **1**, 1996, 1–18.
14. Ignatov, Tz., V. Kaishev. Two-Sided Bounds for the Finite Time Probability of Ruin. *Scand. Actuarial J.*, 1998 (submitted).

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