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MATHEMATICAL MODELING OF THE MELT HEATING PROCESSES IN A FURNACE, PRODUCING FLAT GLASS

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The aim of this paper is to present a two-dimensional model of heat-transfer and transport processes in a glass melting furnace and to use this model for investigation of specifical temperature regimes for different heat flows as well as for the different effective thermal conductivity functions. The mathematical model is elaborated on the base of the real flat glass furnace working in Diamond Ltd in Razgrad. The appropriate numerical methods and their performance are discussed as well.

Keywords: Navier–Stokes equations, melt heating, numerical processes

1991/95 Math. Subject Classification: 76M20

1. INTRODUCTION

Processes taking place in a glass melting furnace producing flat glass are very complicated. In fact, there are five relatively separated physic-chemical processes — silication, refining fusion, degassing, homogenization and cooling, which are closely interconnected at very high temperature and practically occur simultaneously.

The outlet product of the furnace is a glass melt suitable for drawing. Its basic characteristic is its quality, defined by thermal and chemical homogeneity in the drawing volume and in time. Therefore it is very important for the quality of the flat glass that the temperature regime in the furnace be within given limits. Hence the automatic control of glass' quality is directly connected with the control of temperature distribution within the furnace.

The measurement of the temperature of the glass melt is very difficult however. The glass surface temperature is measured using pyrometers and the temperature of melt near the walls and the bottom is usually measured using very expensive special thermocouples. As a matter of fact, the temperature within the glass melt cannot be measured properly and it is practically impossible to have reliable information about it. Information for the temperature distribution in the glass melt can be obtained by mathematical modeling of heat transfer and transport phenomena taking place in the furnace. This information can be used for automatic control of the temperature regime in the furnace and for studying the energeting behaviour of the furnace.

The aim of this paper is to present a two-dimensional model of heat-transfer and transport processes in the furnace and to use this model for investigation of the temperature regime in the furnace for different heat flows as well as for the different effective thermal conductivity functions. The mathematical model is elaborated on the base of the real flat glass furnace working in Diamond Ltd in Razgrad. The appropriate numerical methods and their performance are discussed as well.

2. FORMULATION OF THE MATHEMATICAL PROBLEM

2.1. SCHEME OF THE FURNACE

The glass melting furnace is divided into two parts — a burning chamber and a tank. In this paper we will examine only the tank and will take into account the heat flow from the burning chamber to the glass surface as a boundary conditions on the melt glass surface.

The scheme of the tank and its geometric parameters and coordinate system are given in Fig. 1. The tank consists of two parts — a melting zone (I) and a cooling zone (II). The area HI (Fig. 1) is covered by a batch wedge with a small opening angle. Its length is also given in Fig. 1. The batch material is feed from the doghouse into the furnace with a given temperature.

The side below the batch wedge has a constant temperature which equals the melting temperature (Table 2). The melt batch enters the tank in this place with a given constant velocity v_0. All the heat flow towards the batch in the zone IH is spent for its melting. That is why the heat flow to the glass melt in this zone in fact is equal to zero. In the zone HG the glass melt is heated to the needed temperature for the chemical processes — silication, refining fusion, degassing and homogenization temperature. The glass melt is slightly cooled to the drawing temperature in the cooling zone (II). The homogeneous glass melt is drawn from four drawing machines. The place of the drawing machines is in the end of the cooling zone (area CK in Fig. 1).

2.2. BASIC EQUATIONS

On the base of physical properties of the melt we assume that the glass melt is incompressible Newtonian fluid and the process is steady [1, 2]. The mathematical

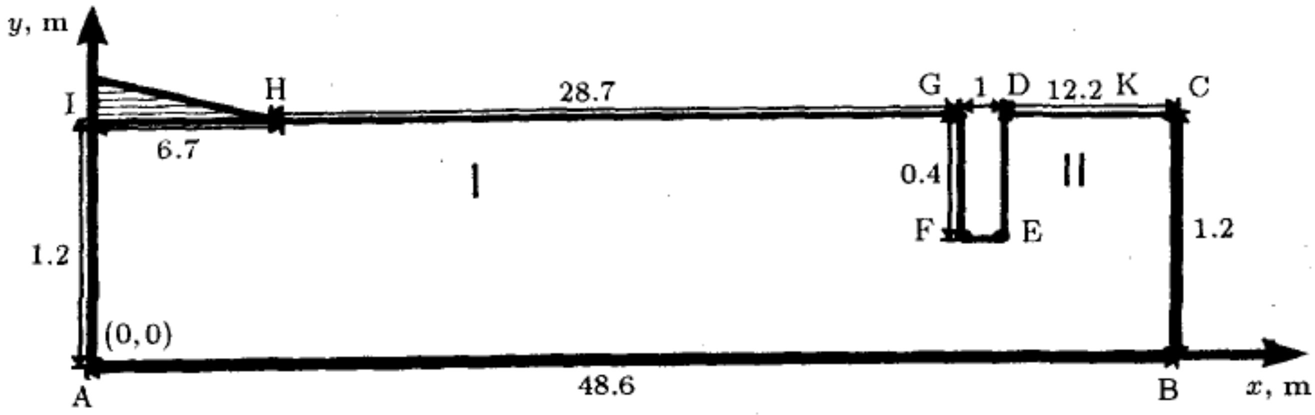

Fig. 1. Scheme of the glass melting furnace and coordinate system

model of the motion and the thermal behaviour of the glass melt are described by the mass, momentum and energy conservation equations that can be expressed as

$$\begin{aligned} &\operatorname{div} \boldsymbol{v} = 0, \\ &\rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla P - \nabla \cdot \tau + \rho\beta(T - T_0)g, \\ &\rho C_p \boldsymbol{v} \cdot \nabla T = \nabla \cdot K_{\text{eff}} \nabla T, \end{aligned} \tag{1}$$

where ρ is the glass density, $\boldsymbol{v}(u, v)$ — the velocity vector with its rectangular coordinates, P — the pressure, τ — the viscous stress tensor, β — the volumetric coefficient of expansion, T and T_0 are the glass temperature and its reference value, g is the gravity acceleration, C_p — the specific heat, and K_{eff} — the effective thermal conductivity, which is a temperature function.

This system can be written in a dimensionless form:

$$\begin{aligned} &\operatorname{div} \boldsymbol{v} = 0, \\ &(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla p + \frac{1}{\text{Re}} \Delta \boldsymbol{v} + \widetilde{F}, \\ &\boldsymbol{v} \cdot \nabla\theta = \left(\frac{1}{\text{Re}} \nabla\right) \cdot \frac{1}{\text{Pr}} \nabla\theta, \end{aligned} \tag{2}$$

where θ, p and $\boldsymbol{v}$ are resp. dimensionless temperature, pressure and velocity; $\widetilde{F}$ is a function of the temperature; Re is the Reynolds number $\text{Re} = L_1 \text{v}_0/\nu$, L_1 and v_0 are resp. the scales of length and velocity, Pr is the Prandtl number $\text{Pr} = \nu/a$, ν is the kinematic viscosity, $a = K_{\text{eff}}/(pC_p)$ is the coefficient of temperature conductivity. Therefore the Prandtl number is a function of the temperature.

The Navier–Stokes equations can be written in an equivalent form, without pressure p, which is more suitable for numerical calculations, namely,

$$\begin{aligned} &u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\text{Re}} \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + \widetilde{F}, \\ &\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \omega. \end{aligned} \tag{3}$$

Here ψ is the stream-function and ω is the vorticity:

$$\omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}. \tag{4}$$

In turn, the components of the velocity vector u and v in a rectangular coordinate system (Fig. 1) can be written as functions of the stream function:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{5}$$

2.3. BOUNDARY CONDITIONS FOR THE TEMPERATURE AND THE STREAM-FUNCTION

The boundary conditions for the temperature and stream-function are given in Table 1. The special feature of Navier–Stokes equations is that the boundary conditions are given only for the stream-function and for the vorticity they must be calculated on the base of the values of the stream-function on the boundaries. The boundary conditions are described in details in [4].

Table 1

	Dimensionless Temperature θ	Dimensionless Stream-function ψ
Front wall; IA	$\frac{\partial \theta}{\partial x} = \frac{U_{w_1} L_0}{K_{\text{eff}}}(\theta - \theta_a)$	$\psi = 0; \quad \frac{\partial \psi}{\partial x} = 0$
Back wall; BC	$-\frac{\partial \theta}{\partial x} = \frac{U_{w_2} L_0}{K_{\text{eff}}}(\theta - \theta_a)$	$\psi = 0; \quad \frac{\partial \psi}{\partial x} = 0$
Bottom; AB	$\frac{\partial \theta}{\partial y} = \frac{U_b L}{K_{\text{eff}}}(\theta - \theta_a)$	$\psi = 0; \quad \frac{\partial \psi}{\partial y} = 0$
Shield assembly wall; DE	$\frac{\partial \theta}{\partial x} = 0$	$\psi = C_1 = \text{const}; \quad \frac{\partial \psi}{\partial x} = 0$
Shield assembly wall; EF	$\frac{\partial \theta}{\partial y} = 0$	$\psi = C_1 = \text{const}; \quad \frac{\partial \psi}{\partial y} = 0$
Shield assembly wall; FG	$\frac{\partial \theta}{\partial x} = 0$	$\psi = C_1 = \text{const}; \quad \frac{\partial \psi}{\partial x} = 0$
Top surface; HI	$\frac{\partial \theta}{\partial y} = 0$	$\psi = \int_{L-L_5}^{L} V\,dx; \quad \frac{\partial \psi}{\partial y} = 0$
Top surface; GH	$\frac{\partial \theta}{\partial y} = q_T \frac{L_0}{K_{\text{eff}} T_0}$	$\psi = C_1 = \text{const}; \quad \frac{\partial \psi}{\partial y} = 0$
Top surface; KD	$\frac{\partial \theta}{\partial y} = q_C \frac{L_0}{K_{\text{eff}} T_0}$	$\psi = C_1 = \text{const}; \quad \frac{\partial \psi}{\partial y} = 0$
Place of drawing machines; CK	$\frac{\partial \theta}{\partial y} = q_C \frac{L_0}{K_{\text{eff}} T_0}$	$\psi = -\int_{0}^{L_1} V\,dx; \quad \frac{\partial \psi}{\partial y} = 0$

The following notations are used in Table 1: U_i is the heat transfer coefficient of solid surfaces (the front wall, the back wall and the bottom), whose values are different for the different surfaces; θ_a is the dimensionless ambient temperature; q_T, q_C are the heat flows, entering the glass melt surface.

2.4. HEAT FLOW AT THE GLASS SURFACE

For the present purposes, we shall formulate a model for the heat flow, using experimental data for the distribution of the fuel flux from the burners. The heat losses due to outlet combustion gasses are taken into account. The distribution of the heat flow entering the glass melt surface along the x-direction and its approximation are given in Fig. 2. The distribution is approximated by the function

$$\log q = 1.0489 + 0.0173x - 0.0000770258x^2. \qquad (6)$$

In the model 14 different functions are to be chosen. The criterion for the best choice is a maximal correlation coefficient and a minimal mean square error.

3. NUMERICAL PROCEDURE

A 5-point approximation is used for the solution of the system of partial differential equations. The grid is non-uniform and it is concentrated in critical areas (near the walls, the bottom and the top surface). The number of points used in the numerical solution is 497 in the x-direction and 23 in the y-direction.

An alternating direction implicit method is used for numerical calculation of the Navier – Stokes equations written in term of stream-function and vorticity and of heat transfer equation. This algorithm is described in detail in [3]. This method is iterative and is based on the solving of the tri-diagonal matrix.

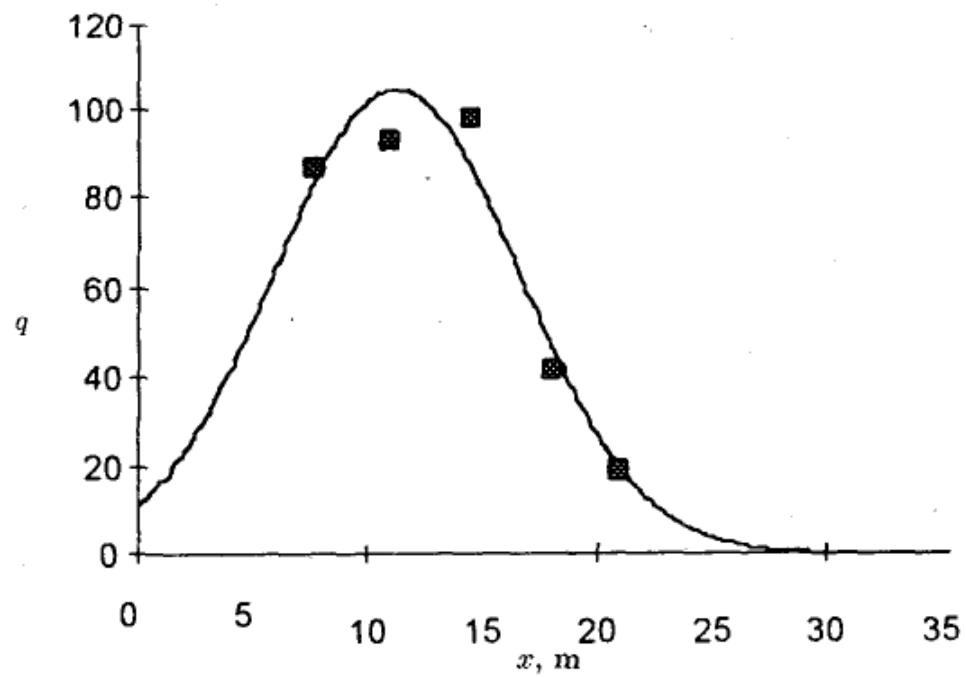

Fig. 2. Distribution of the heat flow in the x-direction and its approximation

4. NUMERICAL RESULTS AND DISCUSSION

We shall use the present mathematical model for an investigation of the heat regime in the glass melt and the influence of the heat flow upon the temperature distribution. Usually, the temperature of the glass melt which enters into the cooling zone is controlled by a change of the fuel flux from the last two couples of burners. That is why we shall study first of all the influence of the heat flow from these burners upon the temperature of the glass melt.

4.1. MODEL PARAMETERS

The tank size considered (Fig. 1) has 48.6 m length and 1.2 m depth. The geometrical dimensions of the melting and cooling zones of the tank are given in Table 2. The heat transfer coefficients of the bottom and front and back walls are given in the same table. The thermophisycal properties for the glass melt (density, specific heat, kinematical viscosity and effective thermal conductivity) for the flat glass melt are taken from the literature [1, 2] and they are summarized in Table 2.

Table 2

Parameters	Value
Density, ρ	2320 kg/m^3
Specific heat, C_p	1256 J/(kg.K)
Kinematics viscosity, ν	0.0101 m^2/s
Effective thermal conductivity, K_{eff}	$5.386 - 2.168 \times 10^{-2}T + 2.058 \times 10^{-5}T^2$
Prandtl number, Pr	$29430.6/K_{eff}$
Reynolds number, Re	0.0222
Melting temperature, T_m	1100 K
Ambient temperature, T_a	350 K
Heat transfer coefficient of the walls and the bottom, U_1	4 $W/(m^2K)$
Melting zone: length, IG	35.4 m
depth, IA	1.2 m
Length of the batch, IH	0.3 m
Cooling zone: length, CD	12.2 m
depth, BC	1.2 m
Shield assembly: length, EF	1.0 m
depth, GF	0.35 m

4.2. RESULTS FOR THE BASIC SIMULATION

The basic simulation is calculated for the parameters given in Table 2 and for the heat flow approximation, given by the function (6) (see Fig. 2). The temperature distribution for the basic simulation is plotted in Fig. 3 and the stream-function field is shown in Fig. 4.

The maximal temperature is calculated at the top surface in the melting zone for $x = 11.1$ m and it equals 2020.4 K. The maximal temperature gradient is in the same area. The minimal temperature in the tank is 1262 K and it is calculated near the front wall and the bottom. The maximal temperature gradient is in the area with maximal heat flow ($x = 11$ m). In this area the difference between temperature at the top and at the bottom is 600 K. The same difference in the area with minimal heat flow ($x = 0$ m) is 50 K. The temperature gradient in the cooling zone near the back wall is only 20 K.

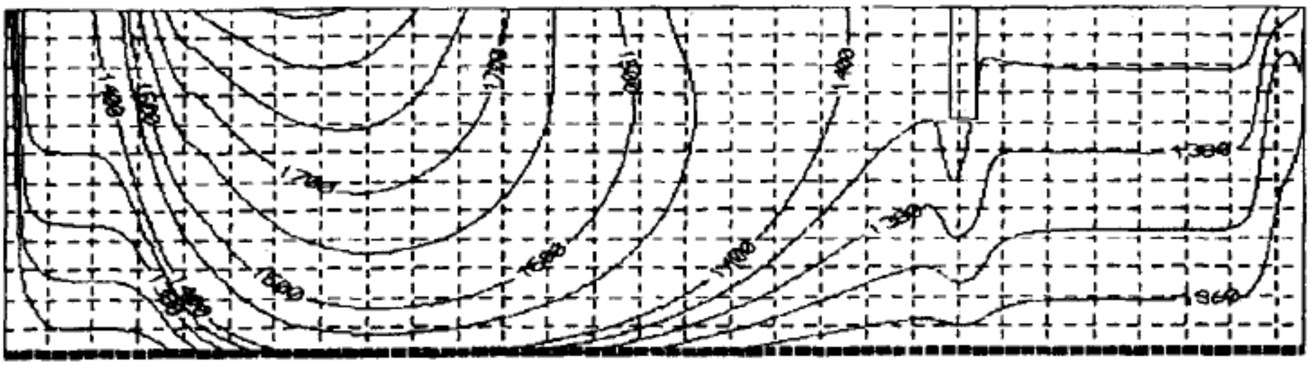

Fig. 3. Temperature distribution for the basic simulation

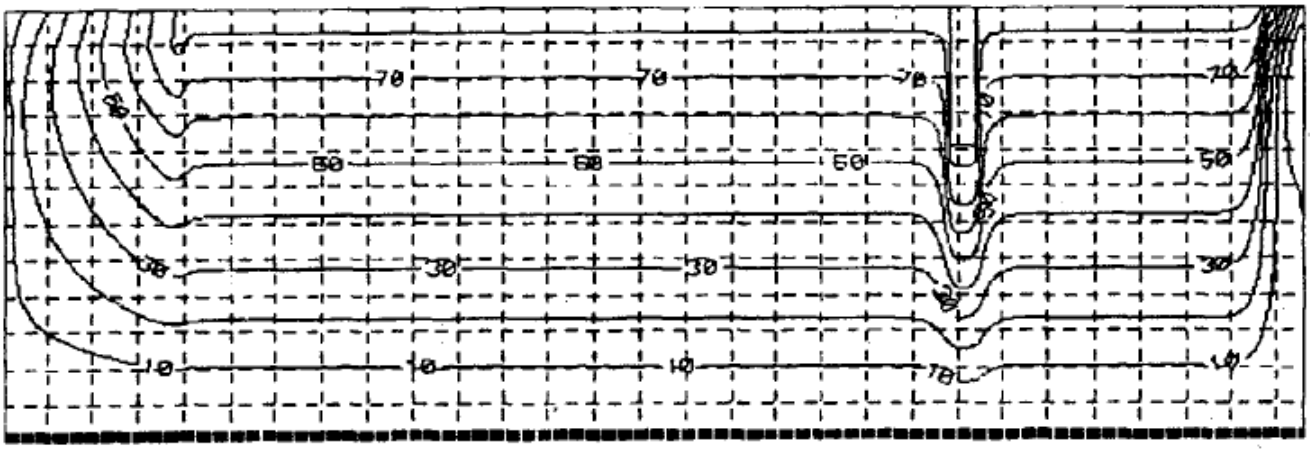

Fig. 4. Stream-function field for the basic simulation

4.3. INVESTIGATION OF THE INFLUENCE OF THE HEAT FLOW

We shall investigate the decrease and the increase of the heat flow from the last two couples of burners by 10 and 20 percent. Different variants for the distribution of the heat flow and its approximation are given in Table 3.

Table 3

Variant	Heat flow from V-th couple of burners, %	Heat flow from VI-th couple of burners, %	Approximation of the heat flow
1	100	100	$\log q = 1.0489 + 0.0173x - 0.0000770258x^2$
2	100	90	$\log q = 0.9695 + 0.018744x - 0.000082906x^2$
3	100	80	$\log q = 0.8806 + 0.020343x - 0.000089479x^2$
4	90	90	$\log q = 1.0272 + 0.017879x - 0.000080401x^2$
5	80	90	$\log q = 1.0918 + 0.016913x - 0.000077602x^2$
6	90	80	$\log q = 0.9384 + 0.019479x - 0.000086975x^2$
7	80	80	$\log q = 1.0030 + 0.018513x - 0.000084176x^2$
8	90	100	$\log q = 1.1067 + 0.016486x - 0.000074522x^2$
9	80	100	$\log q = 1.7113 + 0.015482x - 0.000071723x^2$

The decreasing of the heat flow from the last couple of burners (VI) by 10 percent leads to decrease of the temperature of the glass melt surface with 12 K. 20 percent decreasing of the heat flow leads to decrease of the temperature with 24 K (see Fig. 5). The maximal change of the temperature is near the top surface in the zone under the VI-th couple of burners ($x = 17$ to 25m). Little changes of the fuel flux can lead to a smooth change of the temperature of this zone.

The influence of the heat flow from the V-th couple of burners upon the temperature is given in Fig. 6. This couple of burners dislocates at $x = 17.9$ m. That is why the maximal differences in the temperatures are observed for $x = 15$ m and they are 12.3 K for 10 percent decreasing of the heat flow and 25.7 K for 20 percent decreasing.

Maximal temperature differences when the heat flow decreases from V-th and VI-th couples of burners (variants 1, 4, 5, 6 and 7) are observed for $x = 15$ m to 25 m, too. The decreasing of the heat flow from the V-th and VI-th couples of burners leads to decrease of the temperature at the top surface maximum with 36 K and its temperature difference is under the VI-th couple of burners ($x = 19$ m). Therefore the change of the heat flow from the V-th and VI-th couples of burners can be used for the automatic control of the temperature distribution at the end of the melting zone and in the cooling zone.

4.4. COMPARISON BETWEEN THE COMPUTED RESULTS AND THE EXPERIMENTAL DATA FOR THE TEMPERATURE

The results from the mathematical model could be compared with measurements in the working furnace which is described in this study. The measurement of the temperature of the glass melt is very difficult. The glass surface temperature

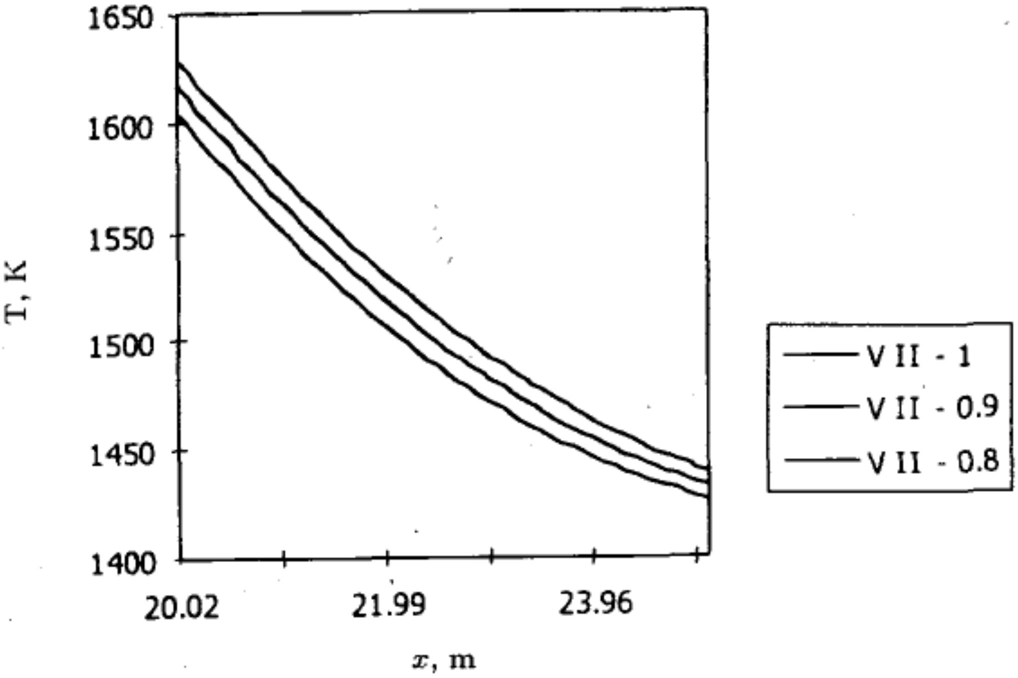

Fig. 5. Temperature of the glass melt at the top surface for variants 1, 2 and 3 (Table 3)

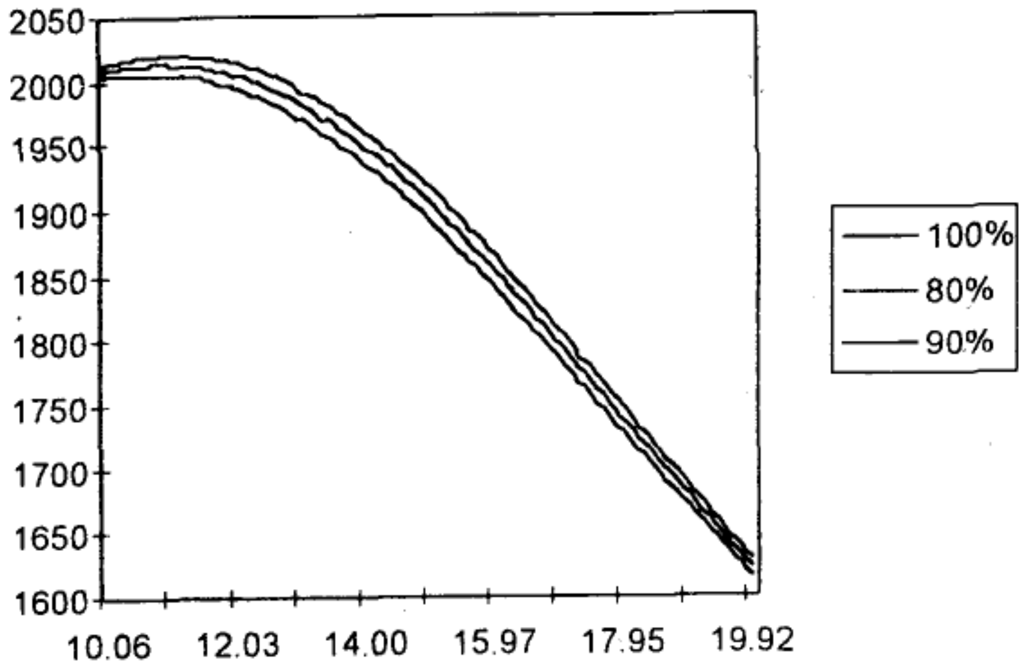

Fig. 6. Temperature of the glass melt at the top surface for variants 1, 8 and 9 (Table 3)

is measured using pyrometers, which leads to considerable errors. As a matter of fact, the temperature within the glass melt cannot be measured and it is practically impossible to have reliable information about it.

The experimental data and computed results for the basic variant for the surface temperature are shown in Fig. 7. It is well seen that the results for the surface temperature agree very well with the measured temperature. The comparison of the simulation results and real data shows that the model gives good results for

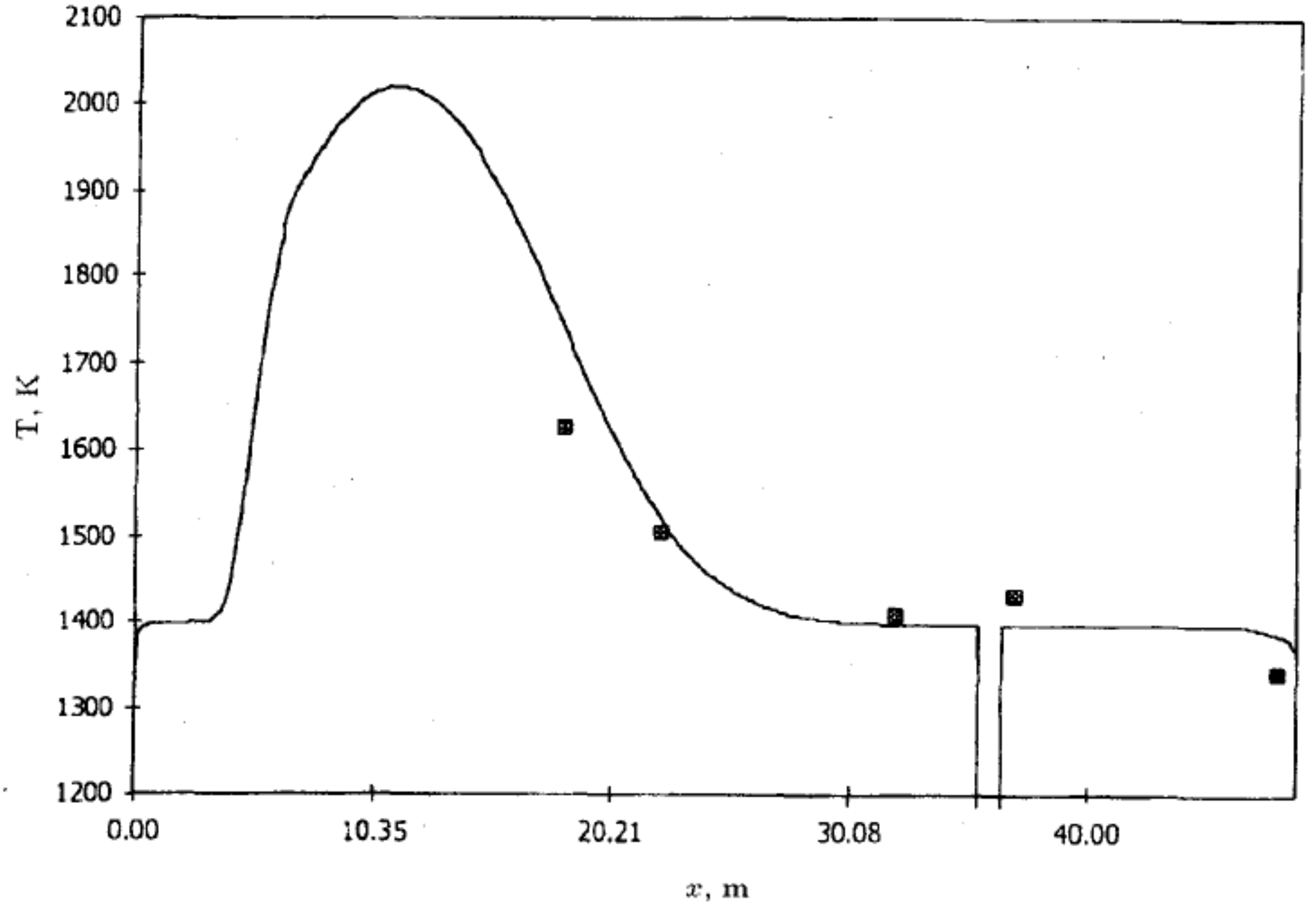

Fig. 7. Computed results and experimental data for the surface temperature

the surface temperature in the melting and cooling zones. The maximal difference is in the zone of burners.

5. CONCLUDING REMARKS

A simplified, but effective mathematical model of the heat transfer and transport processes in the glass melting furnace is presented. The numerical solution uses a finite differences method. The influence of the heat flow from the last two couple of burners is investigated. The comparison of the calculated results and the real data demonstrates a good agreement. The present model offers as well the possibility of computing the appropriate spatial distributions of the temperature and velocity fields. It is possible to study, in particular, the energetical behaviour of the furnace and the influence of its technological parameters upon the temperature and velocity distributions. The approach presented here can be also modified in order to include more specific details of the heat flow and heat transfer in the combustion chamber for calculating more precisely the boundary conditions and the temperature on the glass melting surface. For example, if the air-bubbles motion is taken into account, we can obtain more precise results for the temperature and velocity fields within the furnace.

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