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A SEMANTICS OF LOGIC PROGRAMS WITHOUT SEARCHING*

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A generalized version of the declarative semantics of Horn clause programs on abstract structures is presented. The main feature of the semantics is that it does not admit searching in the domain of the structure.

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1. INTRODUCTION

In this paper we represent and study a semantics of logic programs on abstract structures. A key feature for this semantics is that it does not admit searching in the domain of the structure. We consider partial abstract structures with enumerable domain. The main result is that the class of sets definable by logic programs (LP-definable sets) coincides with the class of domains of Fridman functions in the structure in some fixed point.

In order to prove this, we introduce several auxiliary terms. The names of these terms and the relations between them are given on Fig. 1. An arrow between two terms means that the first one implies the second one. Each arrow is labelled by the number of the proposition where the corresponding implication is proved.

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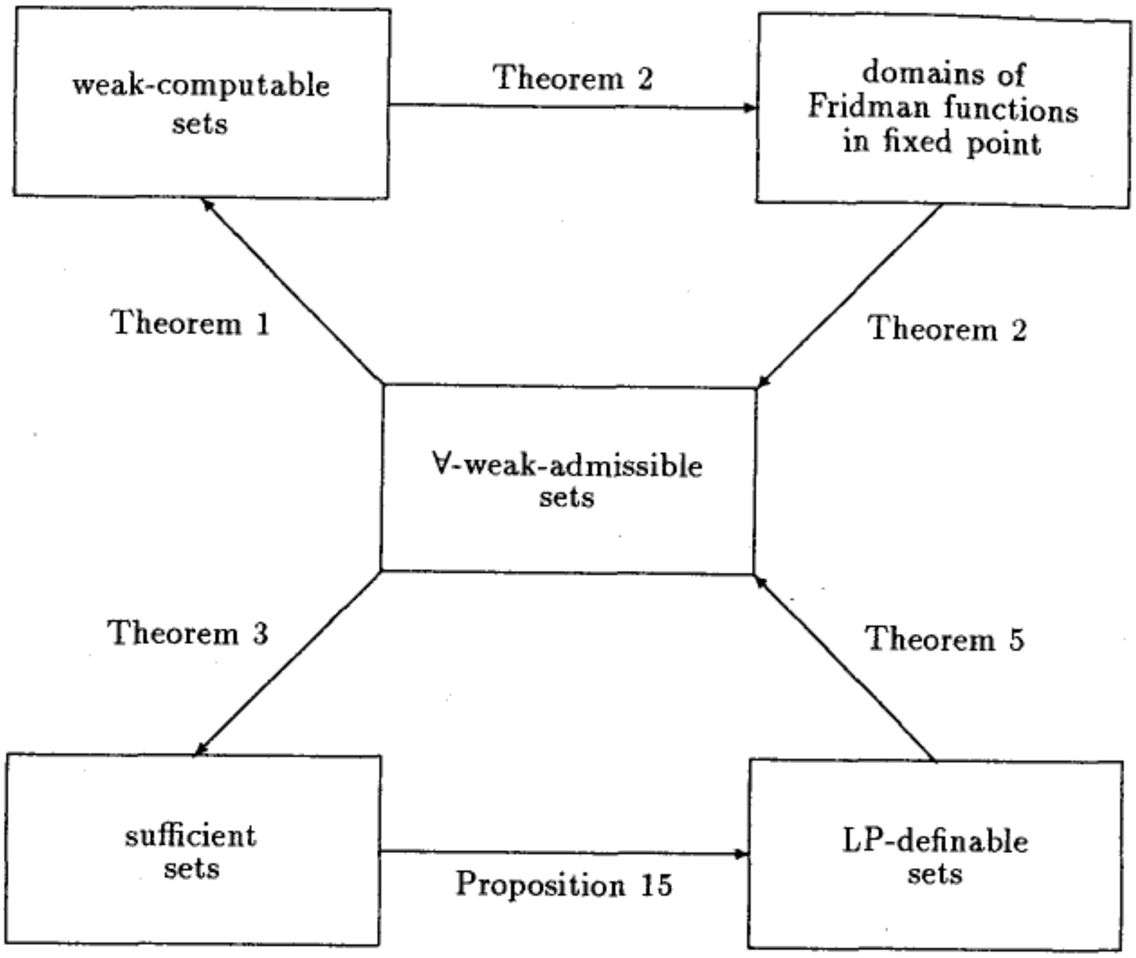

Fig. 1. Relations between the introduced terms

In Section 2 we introduce some basic notions needed in our considerations. In Section 3 we define standard enumerations, which are the main tool in proving all results in the paper. In Section 4 we prove the upper circle in Fig. 1 and in sections 5 and 6 the lower circle is proved.

For the sake of simplicity we consider only structures with unary functions, predicates and parameters. All definitions and results can be easily generalized for functions, predicates and parameters of arbitrary finite arity.

2. PRELIMINARIES

Let $\mathfrak{A} = (B; \theta_1, \dots, \theta_n; \Sigma_0, \dots, \Sigma_k)$ be a partial structure, where the domain of the structure B is a denumerable set, $\theta_1, \dots, \theta_n$ are partial functions of one argument on B, $\Sigma_0, \dots, \Sigma_k$ are partial predicates of one argument on B, $\Sigma_0 = \lambda s.true$ and $n, k \geq 0$. Let $\mathfrak{B} = (N; \varphi_1, \dots, \varphi_n; \sigma_0, \dots, \sigma_k)$ be a partial structure over the set N of the natural numbers. A subset W of N is said to be *recursively enumerable* (r. e.) in $\mathfrak{B}$ iff $W = \Gamma(\varphi_1, \dots, \varphi_n; \sigma_0, \dots, \sigma_k)$ for some enumeration operator Γ (see [1]).

An *enumeration* of the structure $\mathfrak{A}$ is any ordered pair $\langle \alpha, \mathfrak{B} \rangle$, where $\mathfrak{B} = (N; \varphi_1, \dots, \varphi_n; \sigma_0, \dots, \sigma_k)$ is a partial structure, $\sigma_0 = \lambda s.true$, and α is a partial surjective mapping of N onto B such that the following conditions hold:

(i) The domain of α ($Dom(\alpha)$) is closed with respect to the partial operations $\varphi_1, \ldots, \varphi_n$;

(ii) $\alpha(\varphi_i(x)) \simeq \theta_i(\alpha(x))$ for all x of $Dom(\alpha)$, $1 \leq i \leq n$;

(iii) $\sigma_j(x) \Leftrightarrow \Sigma_j(\alpha(x))$ for all x of $Dom(\alpha)$, $1 \leq j \leq k$.

We shall assume that an effective monotonic coding of finite sequences and sets of natural numbers is fixed. If $a_0, \ldots, a_m$ is a sequence of natural numbers, by $\langle a_0, \ldots, a_m\rangle$ we shall denote the code of the sequence $a_0, \ldots, a_m$ and by E_v — the finite set with code v. We shall use the following notations. The letters s, t, p will denote elements of B; x, y, z, u, v will be elements of N. We shall identify the predicates with partial mappings taking values 0 (for "true") and 1 (for "false").

Let $\langle\alpha, \mathfrak{B}\rangle$ be an enumeration of $\mathfrak{A}$. We shall call the set

$$D(\mathfrak{B}) = \{\langle i, x, y\rangle : 1 \leq i \leq n \;\&\; \varphi_i(x) \simeq y\} \cup \{\langle j, x, \varepsilon\rangle : n+1 \leq j \leq n+k \;\;\&\; \sigma_{j-n}(x) \simeq \varepsilon \;\&\; \varepsilon \in \{0,1\}\}$$

a *code* of the structure $\mathfrak{B}$. It is clear that for each $W \subseteq N$, W is r. e. in $\mathfrak{B}$ iff W is r. e. in $D(\mathfrak{B})$.

Let $A \subseteq B$. The set A is called *weak-admissible* in enumeration $\langle\alpha, \mathfrak{B}\rangle$ iff for some r. e. in $\mathfrak{B}$ subset W of N the following conditions hold:

(*) $W \subseteq Dom(\alpha)$;

(**) $\alpha(W) = A$.

A subset A of B is called $\forall$-*weak-admissible* in $\mathfrak{A}$ iff it is weak-admissible in every enumeration $\langle\alpha, \mathfrak{B}\rangle$ of $\mathfrak{A}$.

The equivalence between $\forall$-weak-admissible sets and the sets definable by logic programs will be considered. The $\forall$-weak-admissible sets have an explicit characterization which simplifies the considerations.

Let $\mathfrak{L} = (f_1, \ldots, f_n; T_0, \ldots, T_k)$ be the first-order language corresponding to the structure $\mathfrak{A}$, where $f_1, \ldots, f_n$ are functional symbols, $T_0, \ldots, T_k$ are symbols for predicates, T_0 represents the total predicate $\Sigma_0 = \lambda s.0$.

Let $\{Z_1, Z_2, \ldots\}$ be a denumerable set of variables. We shall use the capital letters X, Y, Z to denote the variables.

If τ is a term of the language $\mathfrak{L}$, then we shall write $\tau(\overline{Z})$ to denote that all of the variables in τ are among $\overline{Z} = (Z_1, \ldots, Z_a)$. If $\tau(\overline{Z})$ is a term and $\bar{t} = t_1, \ldots, t_a$ are arbitrary elements of B, then by $\tau_{\mathfrak{A}}(\overline{Z}/\bar{t})$ we shall denote the value, if it exists, of the term τ in the structure $\mathfrak{A}$ over the elements $t_1, \ldots, t_a$.

Termal predicates in the language $\mathfrak{L}$ are defined by the following inductive clauses:

(i) $T_j(\tau)$, $0 \leq j \leq k$, where τ is a term, are termal predicates;

(ii) If Π is a termal predicate, then $\neg\Pi$ is a termal predicate;

(iii) If Π^1 and Π^2 are termal predicates, then $\Pi^1 \& \Pi^2$ is a termal predicate.

Let $\Pi(\overline{Z})$ be a termal predicate and $t_1, \ldots, t_a$ be arbitrary elements of B. The value $\Pi_{\mathfrak{A}}(\overline{Z}/\bar{t})$ is defined as follows:

(i) If $\Pi = T_j(\tau)$, $0 \le j \le k$, then $\Pi_{\mathfrak{A}}(\overline{Z}/\overline{t}) \simeq \Sigma_j(\tau_{\mathfrak{A}}(\overline{Z}/\overline{t}))$;

(ii) If $\Pi = \Pi^1 \& \Pi^2$, where Π^1 and Π^2 are termal predicates, then

$$\Pi_{\mathfrak{A}}(\overline{Z}/\overline{t}) \simeq \begin{cases} \Pi^2_{\mathfrak{A}}(\overline{Z}/\overline{t}), & \text{if } \Pi^1_{\mathfrak{A}}(\overline{Z}/\overline{t}) \simeq 0, \\ 1, & \text{if } \Pi^1_{\mathfrak{A}}(\overline{Z}/\overline{t}) \simeq 1, \\ \text{undefined} & \text{otherwise.} \end{cases}$$

Let Π be a termal predicate and τ be a term. Then the term $(\Pi \supset \tau)$ is called *conditional term*. Let $Q = (\Pi \supset \tau)$ be a conditional term with variables among $X_1, \ldots, X_a$ and let $s_1, \ldots, s_a$ be arbitrary elements of B. A value $Q_{\mathfrak{A}}(\overline{X}/\overline{s})$ is defined as follows:

$$Q_{\mathfrak{A}}(\overline{X}/\overline{s}) \simeq t \;\Leftrightarrow\; (\Pi_{\mathfrak{A}}(\overline{X}/\overline{s}) \simeq 0 \;\&\; \tau_{\mathfrak{A}}(\overline{X}/\overline{s}) \simeq t).$$

Let fix an effective coding of expressions of $\mathfrak{L}$. The subset A of B is called *weak-computable* iff for some r. e. set V of codes of conditional terms $\{Q^v\}_{v \in V}$ with variables among $Z_1, \ldots, Z_r$ and for fixed elements $t_1, \ldots, t_r$ of B it is true that

$$s \in A \;\Leftrightarrow\; \exists v(v \in V \;\&\; Q^v_{\mathfrak{A}}(\overline{Z}/\overline{t}) \simeq s).$$

3. STANDARD ENUMERATIONS

In order to characterize the LP-definable sets in abstract structures, we shall examine their prototypes in the enumerations of the structures. For this purpose it is enough to restrict our considerations only to a special class of enumerations called standard ones (see [4]). In this section we briefly introduce some definitions and properties of standard enumerations.

Let φ_i^*, $1 \le i \le n$, be the unary recursive function $\lambda x.\langle i, x\rangle$, let $N^0 = N \setminus (Range(\varphi_1^*) \cup \ldots \cup Range(\varphi_n^*))$ and let α^0 be a partial mapping of N^0 onto B.

The partial mapping α of N onto B is defined by the following inductive clauses:

If $x \in N^0$, then $\alpha(x) \simeq \alpha^0(x)$;

If $x = \langle i, y\rangle$, $\alpha(y) \simeq s$ and $\theta_i(s) \simeq t$, then $\alpha(x) \simeq t$.

To the mapping α corresponds the set N_α of natural numbers defined by:

If $x \in Dom(\alpha^0)$, then $x \in N_\alpha$;

If $x = \langle i, y\rangle$ and $y \in N_\alpha$, then $x \in N_\alpha$.

Let $D_1, \ldots, D_n$ be unary partial predicates in N such that:

$$D_i(x) = \begin{cases} 0, & \text{if } x \notin N_\alpha, \\ 0, & \text{if } x \in N_\alpha \text{ and } \theta_i(\alpha(x)) \text{ is defined,} \\ \text{undefined} & \text{otherwise.} \end{cases}$$

The predicates $D_1, \ldots, D_n$ are used to describe the domains of the standard enumeration functions $\varphi_1, \ldots, \varphi_n$ defined as follows:

$$\varphi_i(x) = \begin{cases} \varphi_i^*(x), & \text{if } D_i(x) \simeq 0, \\ \text{undefined} & \text{otherwise.} \end{cases}$$

It is clear that each φ_i is r. e. in $\{D_i\}$ and each D_i is r. e. in $\{\varphi_i\}$, $1 \leq i \leq n$. Let $\sigma_1, \dots, \sigma_k$ be partial predicates in N satisfying the condition

$$x \in N_\alpha \ \Rightarrow\ \sigma_j(x) \simeq \Sigma_j(\alpha(x)),\ 1 \leq j \leq k.$$

Denote by $\mathfrak{B}$ the partial structure $(N; \varphi_1, \dots, \varphi_n; \sigma_0, \dots, \sigma_k)$. Each enumeration $\langle \alpha, \mathfrak{B} \rangle$ obtained by the method described above is called a *standard enumeration.* The mapping α^0 is called a *basis* of the enumeration $\langle \alpha, \mathfrak{B} \rangle$. It is clear that α^0 and the predicates $\sigma_1, \dots, \sigma_k$ completely determine the enumeration $\langle \alpha, \mathfrak{B} \rangle$.

For each natural x we define $|x|$ as follows:

If $x \in N^0$, then $|x| = 0$;

If $x = \langle i, y \rangle$, $1 \leq i \leq n$, then $|x| = |y| + 1$.

The next properties are proved in detail in [4].

Proposition 1. *Let $\langle \alpha, \mathfrak{B} \rangle$ be a standard enumeration and $1 \leq i \leq n$. Then for each natural x, $\alpha(\langle i, x \rangle) \simeq \theta_i(\alpha(x))$.*

Proposition 2. *For each standard enumeration $\langle \alpha, \mathfrak{B} \rangle$, $Dom(\alpha) \subseteq N_\alpha$.*

Proposition 3. *Let $\langle \alpha, \mathfrak{B} \rangle$ be a standard enumeration and $1 \leq i \leq n$. Then for each natural x, $\alpha(\varphi_i(x)) \simeq \theta_i(\alpha(x))$.*

Proposition 4. *Each standard enumeration is an enumeration of the structure $\mathfrak{A}$.*

Define the unary recursive function g in the following way:

If $x \in N^0$, then $g(x) = x$;

If $x = \langle i, y \rangle$, then $g(x) = g(y)$.

Let $\mathfrak{B}^*$ denotes the structure $(N; \varphi_1^*, \dots, \varphi_n^*)$.

Proposition 5. *There exits an effective way to define for each natural x and each variable Y a term $\tau(Y)$ such that $\tau_{\mathfrak{B}^*}(Y/g(x)) = x$.*

Proposition 6. *Let $\tau(Y)$ be a term and $y \in N$. Then for each standard enumeration $\langle \alpha, \mathfrak{B} \rangle$, $\alpha(\tau_{\mathfrak{B}^*}(Y/y)) \simeq \tau_{\mathfrak{A}}((Y/\alpha(y))$.*

Proposition 7. *There exits an effective way to define for each natural x and each variable Y a term $\tau(Y)$ such that for each standard enumeration $\langle \alpha, \mathfrak{B} \rangle$, $\alpha(x) \simeq \tau_{\mathfrak{A}}((Y/\alpha(g(x)))$.*

Let $\langle \alpha, \mathfrak{B} \rangle$ be a standard enumeration. Denote by $R_{\mathfrak{B}}$ the subset of N with the following definition:

$$\langle j, x, \varepsilon \rangle \in R_{\mathfrak{B}} \ \Leftrightarrow\ ((1 \leq j \leq k)\ \&\ \sigma_j(x) \simeq \varepsilon)$$

or

$$(k + 1 \leq j \leq k + n\ \&\ D_{j-k}(x) \simeq \varepsilon)).$$

It is clear that the set W is r. e. in $\mathfrak{B}$ iff it is r. e. in $R_{\mathfrak{B}}$.

Proposition 8. *There exists an effective way to define for each triple* $u = \langle j, x, \varepsilon \rangle$ *and each variable* Y *an atomic predicate* $\Pi(Y)$ *such that for each standard enumeration* $\langle \alpha, \mathfrak{B} \rangle$

$$g(x) \in Dom(\alpha^0) \Rightarrow (u \in R_{\mathfrak{B}} \Leftrightarrow \Pi_{\mathfrak{A}}(Y/\alpha(g(x))) \simeq 0).$$

4. WEAK-COMPUTABILITY AND WEAK-ADMISSIBILITY

LP-definable sets are not convenient for direct examination. That is why we introduce and characterize ∀-weak-admissible sets which are later proved to coincide with the LP-definable sets. In this section we study the relation between ∀-weak-admissibility, weak-computability and Fridman computability.

Theorem 1. *If* A *is ∀-weak-admissible in* $\mathfrak{A}$, *then* A *is weak-computable.*

Proof. Assume A is not weak-computable. We shall construct a standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ of $\mathfrak{A}$ such that A is not admissible in it.

To define the enumeration, we construct a partial surjective mapping α^0 of N^0 onto B. The mapping α^0 will be constructed by steps. On each step q we define a partial mapping α_q of N^0 onto B, a subset H_q of N^0 and partial predicates $\sigma_1^q, \ldots, \sigma_k^q$ such that:

(i) $Dom(\alpha_q)$ and H_q are finite and disjoint;

(ii) $\alpha_q \leq \alpha_{q+1}$ and $H_q \subseteq H_{q+1}$;

(iii) $\sigma_1^q, \ldots, \sigma_k^q$ are partial recursive and defined exactly for those natural y for which $g(y) \in H_q$;

(iv) $\sigma_j^q \subseteq \sigma_j^{q+1}$, $1 \leq j \leq k$.

We take $\alpha^0 = \bigcup_{q=0}^{\infty} \alpha_q$.

With the even steps we ensure that $Range(\alpha^0) = B$. With the odd steps $q = 2n + 1$ we ensure that if Γ_n is the n-th enumeration operator and

$$\langle \alpha, \mathfrak{B} = (N; \varphi_1, \ldots, \varphi_n; \sigma_1, \ldots, \sigma_k) \rangle$$

is a standard enumeration such that $\alpha_q \leq \alpha$, $H_q \cap Dom(\alpha) = \varnothing$, $\sigma_j^q \leq \sigma_j$, $1 \leq j \leq k$, then for $W = \Gamma(R_{\mathfrak{B}})$ at least one of the conditions (*) and (**) fails.

Let $s_0, s_1, \ldots$ be an arbitrary enumeration of B and $x_0, x_1, \ldots$ be an enumeration of N^0, $\alpha_0(x_0) = s_0$ and $\alpha_0(x)$ be undefined for $x \neq x_0$. Let $H_0 = \varnothing$ and each of $\sigma_1^0, \ldots, \sigma_k^0$ be the totally undefined predicate. Let $q > 0$ and α_r, H_r be $\sigma_1^r, \ldots, \sigma_k^r$ defined for $r < q$. We have to consider the following two cases:

I. $q = 2n$. Let z be the first element of the sequence $x_0, x_1, \ldots$ which does not belong to $Dom(\alpha_{q-1}) \cup H_{q-1}$, and s be the first element of the sequence $s_0, s_1, \ldots$ which does not belong to $Range(\alpha_{q-1})$. If such s does not exist, then let s be an arbitrary element of B. Define $\alpha_q(z) \simeq s$ and $\alpha_q(x) \simeq \alpha_{q-1}(x)$ for $x \neq z$, $H_q = H_{q-1}$ and $\sigma_j^q = \sigma_j^{q-1}$, $1 \leq j \leq k$.

II. $q = 2n+1$. Let E_v be the finite set of natural numbers with code v. The set E_v is called *q-consistent* iff:

(i) each element h of E_v is equal to $\langle j, x, \varepsilon\rangle$ for some $n+1 \leq j \leq n+k$ and $\varepsilon \in \{0,1\}$ or $1 \leq j \leq n$ and $\varepsilon = 0$;

(ii) if $\langle j, x, \varepsilon_1\rangle$ and $\langle j, x, \varepsilon_2\rangle$ belong to E_v, then $\varepsilon_1 = \varepsilon_2$;

(iii) if $\langle j, x, \varepsilon\rangle \in E_v$, $n+1 \leq j \leq n+k$ and $g(x) \in H_{q-1}$, then $\sigma_j^{q-1}(x) \simeq \varepsilon$.

Let Γ_n be the n-th enumeration operator defined by W_n — the n-th r. e. set, that is for each set R of natural numbers

$$x \in \Gamma_n(R) \Leftrightarrow \exists v(\langle v, x\rangle \in W_n \,\&\, E_v \subseteq R).$$

Let $u = \langle v, x\rangle$ be element of W_n, $Dom(\alpha_{q-1}) = \{w_1, \ldots, w_m\}$, and $Z_1, \ldots, Z_m$ be distinct variables. Corresponding to u, we define predicates $\Pi^u(Z_1, \ldots, Z_m)$ and $P^u(Z_1, \ldots, Z_m)$ and a term $\tau^u(Z_1, \ldots, Z_m)$ as follows. If E_v is not q-consistent, then $\Pi^u = P^u = \neg T_0(Z_1)$.

Further we consider the case when E_v is q-consistent. We define Π^u in the following way. If E_v does not contain elements of the form $\langle j, x, \varepsilon\rangle$, such that $g(x) \in Dom(\alpha_{q-1})$, then $\Pi^u = T_0(Z_1)$.

Let $\langle j_1, x_1, \varepsilon_1\rangle, \ldots, \langle j_p, x_p, \varepsilon_p\rangle$ be all elements of E_v such that $g(x_i) \in Dom(\alpha_{q-1})$, $1 \leq i \leq p$, and $\Pi^1(Y_1), \ldots, \Pi^p(Y_p)$ be atomic predicates such that:

If $g(x_i) = w_j$ for some j, $1 \leq j \leq m$, then $Y_i = Z_j$;

For each standard enumeration $\langle \alpha, \mathfrak{B}\rangle$, if $g(x_i) \in Dom(\alpha)$, then $\langle j_i, x_i, \varepsilon_i\rangle \in R_{\mathfrak{B}} \Leftrightarrow \Pi^i_{\mathfrak{A}}(Y_i/\alpha(g(x_i))) \simeq 0$.

Define Π^u to be the conjunction of $\Pi^1(Y_1), \ldots, \Pi^p(Y_p)$. Now we define P^u and τ^u to follow the behavior of x.

If $g(x) \notin Dom(\alpha_{q-1})$, then $P^u = \neg T_0(Z_1)$ and $\tau^u = Z_1$.

If $g(x) \in Dom(\alpha_{q-1})$ and $g(x) = w_j$ for some j, $1 \leq j \leq m$, then let $Y = Z_j$ and $\tau(Y)$ be a term such that for each standard enumeration $\langle \alpha, \mathfrak{B}\rangle$, $\alpha(x) \simeq \tau_{\mathfrak{A}}(Y/\alpha(g(x)))$ holds.

Define $P^u = \Pi^u \& T_0(\tau)$ and $\tau^u = \tau$.

We have described a way to construct the r. e. sets $\{\Pi^u\}_{u \in W_n}$, $\{P^u\}_{u \in W_n}$ and $\{\tau^u\}_{u \in W_n}$ for a given W_n.

Denote $\alpha(w_i)$ by t_i, $1 \leq i \leq m$, and let D be a subset of B such that $s \in D$ iff

$$\exists u(u \in W_n \,\&\, P^u_{\mathfrak{A}}(Z_1/t_1, \ldots, Z_m/t_m) \simeq 0 \,\&\, \tau^u_{\mathfrak{A}}(Z_1/t_1, \ldots, Z_m/t_m) \simeq s).$$

It is clear that D is weak-computable and hence $D \neq A$. There are two possible cases.

Case 1. There exists s, which is an element of B, such that $s \in A$ and $s \notin D$. In this case we have also two possibilities:

a. For some $u \in W_n$, $u = \langle v, x\rangle$, we have

$$\Pi^u_{\mathfrak{A}}(Z_1/t_1, \ldots, Z_m/t_m) \simeq 0 \tag{1}$$

and

$$P^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \not\simeq 0. \tag{2}$$

From (1) and (2) it follows that

$$g(x) \in Dom(\alpha_{q-1}) \;\Rightarrow\; \tau_{\mathfrak{A}}(Y/\alpha(g(x)) \text{ is undefined}. \tag{3}$$

Let

$$L = H_{q-1} \cup \{g(y) \mid \exists j \exists \varepsilon(\langle j,y,\varepsilon\rangle \in E_v \;\&\; g(y) \notin Dom(\alpha_{q-1}))\}.$$

We define $\alpha_q \equiv \alpha_{q-1}$. If $g(x) \in Dom(\alpha_q)$, then define $H_q \equiv L$, else $H_q \equiv H_{q-1} \cup \{g(x)\}$. And, finally, we define σ^q_j, $1 \le j \le k$, by the following clauses:

If $g(y) \notin H_q$, then $\sigma^q_j(y)$ is undefined;

If $g(y) \in H_q \setminus H_{q-1}$, then if $\langle n+j,y,\varepsilon\rangle \in E_v$ we have $\sigma^q_j(y) \simeq \varepsilon$, else $\sigma^q_j(y) \simeq 0$;

If $g(y) \in H_{q-1}$, then $\sigma^q_j(y) \simeq \sigma^{q-1}_j(y)$.

It follows from (1) that E_v is q-consistent and hence $\sigma^q_j(y)$ are correctly defined and $\sigma^{q-1}_j \le \sigma^q_j$.

Let $\langle\alpha,\mathfrak{B}\rangle$ be a standard enumeration such that $\alpha \ge \alpha_q$, $Dom(\alpha) \cap H_q = \varnothing$, $\sigma_j \ge \sigma^q_j$, $1 \le j \le k$, and let $W = \Gamma_n(R_{\mathfrak{B}})$. We shall prove that $E_v \subseteq R_{\mathfrak{B}}$.

Indeed, let $\langle j,y,\varepsilon\rangle \in E_v$. If $g(y) \in Dom(\alpha_q)$, then $g(y) \in Dom(\alpha)$ and (1) yields $\langle j,y,\varepsilon\rangle \in R_{\mathfrak{B}}$. If $g(y) \notin Dom(\alpha_q)$, then $g(y) \in H_q$ and we also have $\sigma_j \ge \sigma^q_j$ and $H_q \cap N_\alpha = \varnothing$, hence $\langle j,y,\varepsilon\rangle \in R_{\mathfrak{B}}$.

Suppose that $x \in Dom(\alpha)$. Then $g(x) \in Dom(\alpha_{q-1})$. From the definition of τ^u we obtain that $\tau^u_{\mathfrak{A}}(Y/\alpha(g(x))) \simeq \alpha(x)$ and $\tau^u_{\mathfrak{A}}(Y/\alpha(g(x)))$ is defined. This contradicts (3), hence $x \notin Dom(\alpha)$, which implies $W \not\subseteq Dom(\alpha)$.

b. For each $u \in W_n$ such that $u = \langle v,x\rangle$, $\Pi^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$ implies $P^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$. In this case let $\alpha_q \equiv \alpha_{q-1}$, $H_q \equiv H_{q-1}$ and $\sigma^q_j \equiv \sigma^{q-1}_j$, $1 \le j \le k$. Let $\langle\alpha,\mathfrak{B}\rangle$ be a standard enumeration such that $\alpha \ge \alpha_q$, $Dom(\alpha) \cap H_q = \varnothing$, $\sigma_j \ge \sigma^q_j$, $1 \le j \le k$, and $W = \Gamma_n(R_{\mathfrak{B}})$. Suppose that there exists $x \in W$ such that $\alpha(x) = s$. Then there exists $u \in W_n$ such that $u = \langle v,x\rangle$ and $E_v \subseteq R_{\mathfrak{B}}$. From the definitions it follows that $\Pi^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$ and hence $P^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$. We obtained $\alpha(x) \simeq \tau^u_{\mathfrak{A}}(Y/\alpha(g(x))) \simeq s$, which contradicts the assumption $s \notin D$. We conclude that $A \not\equiv \alpha(W)$.

Case 2. There exists $s \in B$ such that $s \notin A$ and $s \in D$. This implies the existence of $u \in W_n$ such that $u = \langle v,x\rangle$ and $P^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$ and $\tau^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq s$. Then $\Pi^u_{\mathfrak{A}}(Z_1/t_1,\ldots,Z_m/t_m) \simeq 0$ and hence E_v is q-consistent. Let $\alpha_q \equiv \alpha_{q-1}$ and

$$H_q = H_{q-1} \cup \{g(y) \mid \exists j \exists \varepsilon(\langle j,y,\varepsilon\rangle \in E_v \;\&\; g(y) \notin Dom(\alpha_{q-1}))\}.$$

We define the predicates σ^q_j, $1 \le j \le k$, by the following clauses:

If $g(y) \notin H_q$, then $\sigma^q_j(y)$ is undefined;

If $g(y) \in H_q$, then

$$\sigma^q_j(y) \simeq \begin{cases} \varepsilon, & \text{if } \langle j,y,\varepsilon\rangle \in E_v, \\ 0, & \text{if } g(y) \in H_q/H_{q-1} \text{ and } \langle j,y,\varepsilon\rangle \notin E_v, \\ \sigma^{q-1}_j(y), & \text{if } q(y) \in H_{q-1}. \end{cases}$$

Let $\langle\alpha, \mathfrak{B}\rangle$ be a standard enumeration such that $\alpha \geq \alpha_q$, $Dom(\alpha) \cap H_q = \varnothing$, $\sigma_j \geq \sigma_j^q$, $1 \leq j \leq k$, and $W = \Gamma_n(R_{\mathfrak{B}})$. Analogously to *Case* **1a**, we can prove that $E_v \subseteq R_{\mathfrak{B}}$ and hence $x \in W$. From $P^u_{\mathfrak{A}}(Z_1/t_1, \ldots, Z_m/t_m) \simeq 0$ and from the definition of τ^u it follows that $x \in Dom(\alpha_q)$ and $\alpha(x) \simeq s$. And $s \notin A$ implies $A \not\equiv \alpha(W)$.

Now we are ready to complete the proof defining the required enumeration. Let $\alpha^0 = \bigcup\limits_{q=0}^{\infty} \alpha_q$, $\sigma_j^* = \bigcup\limits_{q=0}^{\infty} \sigma_j^q$, $1 \leq j \leq k$, and $H = \bigcup\limits_{q=0}^{\infty} H_q$. Let $\langle\alpha, \mathfrak{B} = (N; \varphi_1, \ldots, \varphi_n; \sigma_0, \ldots, \sigma_m)\rangle$ be a standard enumeration with basis α^0 and

$$\sigma_j(x) \simeq \begin{cases} \sigma_j^*(x), & \text{if } x \notin N_\alpha, \\ \Sigma_j(\alpha(x)) & \text{otherwise,} \end{cases} \qquad 1 \leq j \leq k.$$

□

Let $F(\mathfrak{A})$ be the class of all Fridman computable functions in $\mathfrak{A}$.

Let $\mathfrak{A}_i = (B_i; \theta_1^i, \ldots, \theta_n^i; \Sigma_0^i, \ldots, \Sigma_k^i)$, $i = 1, 2$, be two partial structures, where the corresponding functions and predicates have the same arity. The mapping κ of B_1 onto B_2 is called a *strong homomorphism* iff:

(i) κ is a surjective mapping;

(ii) $\kappa(\theta_i^1(s_1, \ldots, s_{a_i})) \simeq \theta_i^2(\kappa(s_1), \ldots, \kappa(s_{a_i}))$ for each $(s_1, \ldots, s_{a_i}) \in B_1^{a_i}$, $1 \leq i \leq n$;

(iii) $\Sigma_j^1(s_1, \ldots, s_{b_j}) \Leftrightarrow \Sigma_j^2(\kappa(s_1), \ldots, \kappa(s_{b_j}))$ for each $(s_1, \ldots, s_{b_j}) \in B_1^{b_j}$, $1 \leq j \leq k$.

It is easy to show the following properties of the Fridman computability:

1. *Invariantness.* If κ is a strong homomorphism between $\mathfrak{A}_1$ and $\mathfrak{A}_2$ and $\theta_2 \in F(\mathfrak{A}_2)$, then there exists $\theta_1 \in F(\mathfrak{A}_1)$ (of the same arity) such that $\kappa(\theta_1(s_1, \ldots, s_a)) \simeq \theta_2(\kappa(s_1), \ldots, \kappa(s_a))$ for each $(s_1, \ldots, s_a) \in B_1^a$.

2. *Effectiveness.* If $\mathfrak{A} = (N; \varphi_1, \ldots, \varphi_n; \sigma_0, \ldots, \sigma_k)$ is a partial structure and $\varphi \in F(\mathfrak{A})$, then φ is r. e. in the functions and predicates of $\mathfrak{A}$.

3. *Substructure property.* Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be partial structures such that $B_1 \subseteq B_2$. Let φ_j^1 and Σ_j^1 be the restrictions of φ_j^2 and Σ_j^2 on B_1, $1 \leq j \leq k$, $1 \leq i \leq n$, and $\theta_1 \in F(\mathfrak{A}_1)$. Then there exists $\theta_2 \in F(\mathfrak{A}_2)$ such that $\theta_1(s_1, \ldots, s_a) \simeq \theta_2(s_1, \ldots, s_a)$ for each $(s_1, \ldots, s_a) \in B_1^a$.

Using these properties we are ready to establish a relation between $\forall$-weak-admissibility and Fridman computability.

Theorem 2. *A is $\forall$-weak admissible iff there exists $\theta \in F(\mathfrak{A})$ such that $A \equiv \theta(t_1^0, \ldots, t_r^0)$ for some $(t_1^0, \ldots, t_r^0) \in B^r$.*

Proof. Let $\theta \in F(\mathfrak{A})$ and $A \equiv \theta(t_1^0, \ldots, t_r^0)$ for some $(t_1^0, \ldots, t_r^0) \in B^r$. Let $\langle\alpha, \mathfrak{B}\rangle$ be an arbitrary partial enumeration of $\mathfrak{A}$ and

$$\mathfrak{B}' = (Dom(\alpha); \varphi_1', \ldots, \varphi_n'; \sigma_0', \ldots, \sigma_k'),$$

where φ_i' and σ_j' are the restrictions of φ_i and σ_j on $Dom(\alpha)$, $1 \leq i \leq n$, $0 \leq j \leq k$. Then α is a strong homomorphism between $\mathfrak{B}'$ and $\mathfrak{A}$. Thus there exists $\varphi' \in F(\mathfrak{B}')$ such that φ' is an α-prototype of θ. From the substructure property of Fridman computability, there exists $\varphi \in F(\mathfrak{B})$ such that $\varphi(x_1, \ldots, x_r) \simeq \varphi'(x_1, \ldots, x_r)$ for each $(x_1, \ldots, x_r) \in (Dom(\alpha))^r$. Because of the effectiveness of Fridman computability, φ is r. e. in $\mathfrak{B}$. We obtained that $A \equiv \alpha(\varphi'(x_1, \ldots, x_r))$, $x_1, \ldots, x_r \in Dom(\alpha)$ and $\alpha(x_i) \simeq t_i$, $1 \leq i \leq r$. The set $W = \varphi(x_1, \ldots, x_r)$ is r. e. in $\mathfrak{B}$, because φ is such and $\varphi \geq \varphi'$. Finally, $W \subseteq Dom(\alpha)$ and $A = \alpha(W)$, i. e. A is $\forall$-weak-admissible.

Now let A be $\forall$-weak-admissible. Then from the previous theorem, A is weak-computable. Using the corresponding definitions, we can easily prove the "if" part of the theorem. $\Box$

5. SUFFICIENCY AND WEAK-ADMISSIBILITY

In this section we introduce the notion of sufficiency and establish the relation between sufficiency and weak-admissibility.

Further we assume that the structure $\mathfrak{A} = (B; \theta_1, \ldots, \theta_n; \Sigma_0, \ldots, \Sigma_k)$ is such that the predicates $\Sigma_1, \ldots, \Sigma_k$ take only value 0 (*true*) wherever defined. This assumption is not restrictive, because each predicate Σ can be represented by the following two predicates:

$$\Sigma^\delta(s) \simeq \begin{cases} 0, & \text{if } \Sigma(s) \simeq \delta, \\ \text{undefined} & \text{otherwise,} \end{cases} \qquad \delta = 0, 1.$$

The extra condition we impose is due to the syntax of Horn clause logic programs. The negative information of the structure cannot be used because a negation in clause tails is not allowed. Let fix the structure $\mathfrak{A}$ and modify some of the notions introduced according to the new limitation.

A standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ is called *positive* iff $\sigma_1, \ldots, \sigma_k$ take only value 0 wherever defined. Further all the enumerations are assumed to be positive and thus we can simplify the code of the structure considering the set

$$\langle \mathfrak{B} \rangle = \{\langle j, x \rangle \mid n+1 \leq j \leq n+k \text{ and } \sigma_{j-n}(x) \text{ is defined}\} \cup$$
$$\{\langle i, x \rangle \mid 1 \leq i \leq n \text{ and } D_i(x) \text{ is defined}\}$$

instead of $R_{\mathfrak{B}}$. Note that $\langle \alpha, \mathfrak{B} \rangle$ is a positive standard enumeration and W is r. e. in $\mathfrak{B}$ iff $W = \Gamma(\langle \mathfrak{B} \rangle)$ for some enumeration operator Γ.

The pair $\langle \alpha', H' \rangle$ is called a *finite part* iff:

(i) α' is a finite mapping of N^0 onto B;

(ii) H' is a finite subset of N^0 and $Dom(\alpha') \cap H' = \varnothing$.

The positive standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ *extends* the finite part $\langle \alpha', H' \rangle$ iff:

(i) $\alpha \geq \alpha'$;

(ii) $Dom(\alpha) \cap H' = \varnothing$;

(iii) $\sigma_j(x) \simeq 0$ for each $x \in N$ such that $g(x) \in H'$, $1 \leq j \leq k$.

Let fix an arbitrary finite part $\langle \alpha', H' \rangle$, $Dom(\alpha') = \{w_1, \ldots, w_r\}$ and $\alpha'(w_i) = s_i$, $1 \leq i \leq r$. Let $c_{s_1}, \ldots, c_{s_r}$ be new constants which we shall interpret as names of $s_1, \ldots, s_r$. Further we consider terms and predicates of the first order language $\mathfrak{L} = (c_{s_1}, \ldots, c_{s_r}, f_1, \ldots, f_n, T_0, \ldots, T_k)$.

The finite set with code v, E_v, is called *correct* iff it consists only of elements of the form $\langle j, x \rangle$ for some natural x and $1 \leq j \leq k + n$.

The next propositions are similar to the propositions for the standard enumerations and have straightforward proves.

Proposition 9. *Let $x \in N$ and $g(x) \in Dom(\alpha')$. Then there exists an effective way to define a term τ^x without variables such that each positive standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ extending $\langle \alpha', H' \rangle$ satisfies $\alpha(x) \simeq \tau^x_{\mathfrak{A}}$.*

Proposition 10. *Let $x \in N$, $g(x) \in Dom(\alpha')$ and $1 \leq j \leq n + k$. Then there exists an effective way for $u = \langle j, x \rangle$ to define an atomic predicate Π^u without variables and negations such that for each standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ we have*

$$\langle j, x \rangle \in \langle \mathfrak{B} \rangle \;\Leftrightarrow\; \Pi^u_{\mathfrak{A}} \simeq 0.$$

We shall identify each finite set of atoms without variables with their conjunctions. The empty set we shall identify with the logical constant *true*.

Let E be a correct finite set of naturals. By $\tilde{E}$ we denote the set

$$\{\Pi^u \mid u = \langle j, x \rangle \;\&\; g(x) \in Dom(\alpha') \;\&\; u \in E\}.$$

We shall call the set E *appropriate* for the finite part $\langle \alpha', H' \rangle$) iff it is correct and $\tilde{E}_{\mathfrak{A}} \simeq 0$.

Let W be a r. e. set and Γ be the enumeration operator defined by W. The notion of compatibility of a finite part and an enumeration operator introduced below reflects the fact that in logic programs, where a search in the domain of the structure is not allowed, only a finite information supplied by constants is available, while W contains much more information which is not accessible.

The finite part $\langle \alpha', H' \rangle$ and the enumeration operator Γ are *compatible* iff for each $u = \langle v, x \rangle \in W$, such that E_v is appropriate for $\langle \alpha', H' \rangle$, we have $g(x) \in Dom(\alpha')$.

Let Γ be an enumeration operator and let the finite part $\langle \alpha', H' \rangle$ be compatible to Γ. The subset A of B is called *sufficient* for $\langle \alpha', H' \rangle$ and Γ iff:

(i) For each positive standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ extending $\langle \alpha', H' \rangle$, it is true that $\alpha(\Gamma(\langle \mathfrak{B} \rangle)) \subseteq A$;

(ii) For each $s \in A$ there exists a finite part $\langle \alpha'', H'' \rangle$ such that $\alpha'' \geq \alpha'$, $H'' \supseteq H'$ and for each positive standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ extending $\langle \alpha'', H'' \rangle$ it is true that $s \in \alpha(\Gamma(\langle \mathfrak{B} \rangle))$.

It is easy to show the following

Proposition 11. *For each compatible finite part $\langle \alpha', H' \rangle$ and enumeration operator Γ there exists at most one subset of B which is sufficient for them.*

A class of sets $\mathfrak{P}$ is called *sufficient* iff for each compatible finite part $\langle \alpha', H' \rangle$ and enumeration operator Γ there exists a set $A \in \mathfrak{P}$ which is sufficient for Γ and $\langle \alpha', H' \rangle$.

Theorem 3. *Each sufficient class contains all* $\forall$*-weak-admissible sets.*

Proof. Let $\mathfrak{P}$ be a sufficient class and $A \notin \mathfrak{P}$. We will prove that A is not $\forall$-weak-admissible constructing by steps a positive standard enumeration in which A is not weak-admissible. For the purpose we shall construct a partial surjective mapping α^0 of N^0 onto B and a subset H of N^0 such that $Dom(\alpha^0) \cap H = \varnothing$. The set H defines the predicates $\sigma_1, \ldots, \sigma_k$ out of N_α. Even steps ensure that α^0 is a surjective mapping and the odd steps $q = 2n + 1$ ensure that A is not weak-admissible for the n-th enumeration operator Γ_n.

Let $s_0, s_1, \ldots$ be an arbitrary ordering of elements of B and $x_0, x_1, \ldots$ be an arbitrary ordering of elements of N^0. Let $\alpha_0(x_0) \simeq s_0$ and $\alpha_0(x)$ be undefined otherwise. Let $H_0 = \varnothing$. Now suppose we have defined $\langle \alpha_l, H_l \rangle$, $0 \leq l < q$. We define $\langle \alpha_q, H_q \rangle$ as follows:

I. $q = 2n$. Then let $H_q \equiv H_{q-1}$ and s be the first element of the sequence $s_0, s_1, \ldots$ which is not in $Range(\alpha_{q-1})$ (if there is no such element, let s be an arbitrary element of B). Let x be the first element of the sequence $x_0, x_1, \ldots$ which is not in $Dom(\alpha_{q-1}) \cup H_{q-1}$. Then let $\alpha_q(y) \simeq \alpha_{q-1}(y)$ for each $y \in N^0$, $y \neq x$ and $\alpha_q(x) \simeq s$.

II. $q = 2n+1$. Let Γ_n be the n-th enumeration operator. Consider the following cases:

Case 1. Γ_n and $\langle \alpha_{q-1}, H_{q-1} \rangle$ are incompatible. Let W_n be the r. e. set defining Γ_n. Then there exists $u \in W_n$ such that $u = \langle v, x \rangle$ and E_v is appropriate for $\langle \alpha_{q-1}, H_{q-1} \rangle$ and $g(x) \notin Dom(\alpha_q)$. Let $L = \{g(y) \mid \exists j(\langle j, x \rangle \in E_v \;\&\; g(y) \notin Dom(\alpha_{q-1}))\}$, $H_q = H_{q-1} \cup L \cup \{g(x)\}$ and $\alpha_q \equiv \alpha_{q-1}$. Let $\langle \alpha, \mathfrak{B} \rangle$ be a positive standard enumeration extending $\langle \alpha_q, H_q \rangle$ and $h = \langle j, y \rangle \in E_v$. If $g(y) \in Dom(\alpha_q)$, then $g(y) \in Dom(\alpha_{q-1})$, hence $\Pi^h_{\mathfrak{A}} \simeq 0$ and thus $h \in \langle \mathfrak{B} \rangle$. If $g(y) \notin Dom(\alpha_q)$, then $g(y) \in H_{q-1}$. If $k + 1 \leq j \leq k + n$, then we obtain $h \in \langle \mathfrak{B} \rangle$ from the definition of extension, and if $1 \leq j \leq n$, then we obtain this from the definition of standard enumeration and from $Dom(\alpha^0) \cap H_q = \varnothing$.

In this way we have proved that $E_v \subseteq \langle \mathfrak{B} \rangle$, which implies $x \in \Gamma_n(\langle \mathfrak{B} \rangle)$. From $g(x) \in H_q$ it follows that $x \notin Dom(\alpha)$, that is $\Gamma_n(\langle \mathfrak{B} \rangle) \not\subseteq Dom(\alpha)$ and A is not weak-admissible in $\langle \alpha, \mathfrak{B} \rangle$.

Case 2. Γ_n and $\langle \alpha_{q-1}, H_{q-1} \rangle$ are compatible. Let $D \in \mathfrak{P}$ be the set which is sufficient for Γ_n and $\langle \alpha_{q-1}, H_{q-1} \rangle$. Then $D \not\equiv A$. In this case there are two subcases possible:

a. There exists $s \in B$ such that $s \in A$ and $s \notin D$. Let $\alpha_q \equiv \alpha_{q-1}$ and $H_q \equiv H_{q-1}$. Sufficiency of D implies that for each standard enumeration $\langle \alpha, \mathfrak{B} \rangle$ extending $\langle \alpha_{q-1}, H_{q-1} \rangle$ we have $\alpha(\Gamma_n(\langle \mathfrak{B} \rangle)) \subseteq D$. Then $s \notin \alpha(\Gamma_n(\langle \mathfrak{B} \rangle))$, which means $A \not\equiv \alpha(\Gamma_n(\langle \mathfrak{B} \rangle))$.

b. There exists $s \in B$ such that $s \notin A$ and $s \in D$. In this case there exists a finite part $\langle \alpha', H' \rangle$ extending $\langle \alpha_{q-1}, H_{q-1} \rangle$, that is $\alpha' \geq \alpha_{q-1}$ and $H' \supseteq H_{q-1}$ and for each positive standard enumeration extending $\langle \alpha_{q-1}, H_{q-1} \rangle$ it is true that $s \in \alpha(\Gamma_n(\langle \mathfrak{B} \rangle))$. Let $\alpha_q \equiv \alpha'$ and $H_q \equiv H'$. Then for each standard enumeration extending $\langle \alpha_q, H_q \rangle$ it is true that $\alpha(\Gamma_n(\langle \mathfrak{B} \rangle)) \not\equiv A$.

Finally, let $\alpha^0 = \bigcup_{q=1}^{\infty} \alpha_q$, $H^0 = \bigcup_{q=1}^{\infty} H_q$ and $\sigma_j^* = \bigcup_{q=1}^{\infty} \sigma_j^q$, where

$$\sigma_j^q(x) \simeq \begin{cases} 0, & \text{if } g(x) \in H_q, \\ \text{undefined} & \text{otherwise,} \end{cases} \qquad 1 \leq j \leq k.$$

We define the predicates $\sigma_1, \ldots, \sigma_k$ as follows:

$$\sigma_j(x) \simeq \begin{cases} \Sigma_j(\alpha(x)), & \text{if } x \in N_\alpha, \\ \sigma_j^*(x) & \text{otherwise.} \end{cases}$$

It is clear that the standard enumeration $\langle \alpha, \mathfrak{B} \rangle$, determined by α^0 and $\sigma_1, \ldots,$ σ_k, is positive, correctly defined and it extends the finite parts $\langle \alpha_q, H_q \rangle$, $q = 0, 1, \ldots,$ which means that A is not weak-admissible in $\langle \alpha, \mathfrak{B} \rangle$. □

6. LP-DEFINABILITY AND WEAK-ADMISSIBILITY

In this section we give a formal definition of LP-definable sets and for each compatible enumeration operator Γ and finite part $\langle \alpha', H' \rangle$ we construct a logic program $\langle P, F \rangle$ which defines a set sufficient for them. In this way we prove the equivalence between LP-definability and $\forall$-weak admissibility.

Let fix a structure $\mathfrak{A}$ which predicates are true wherever defined and let $\mathfrak{L} = (f_1, \ldots, f_n; T_0, \ldots, T_k)$ be a first order language corresponding to $\mathfrak{A}$. Let $\mathfrak{L}_C$ be the enrichment of $\mathfrak{L}$ with constants $c_1, \ldots, c_r$ and $\mathfrak{T}_C$ be the set of all terms without variables of $\mathfrak{L}_C$. We denote the set of all atoms of the form $T_j(\tau)$, where $0 \leq j \leq k$, $\tau \in \mathfrak{T}_C$ and $\Sigma_j(\tau_{\mathfrak{A}}) \simeq 0$ (the last means that $T_j(\tau)$ is true in $\mathfrak{A}$) by $\partial^C(\mathfrak{A})$.

Logic programs are called formulae of the form $F^1 \& \ldots \& F^l$, where F^i is an universal closure of Horn clause, i. e. F^i is of the form

$$\forall X_1 \ldots \forall X_r (\Pi \vee \neg \Pi_1 \vee \ldots \neg \Pi_n),$$

where $n \geq 0$ and $\Pi, \Pi_1, \ldots, \Pi_n$ are atomic predicates. We shall use the usual Prolog notation

$$\Pi \text{ :- } \Pi_1, \ldots, \Pi_n.$$

for the Horn clauses. Π is called a *head* and $\Pi_1, \ldots, \Pi_n$ — a *tail* of the clause.

Let F be a new predicate symbol which is not among $T_0, \ldots, T_k$. For the sake of simplicity F is assumed to be an unary predicate symbol. All the definitions and proofs can be easily generalized for the case of a higher arity.

By $\mathfrak{L}_p$ we denote the language of the logic program P. The symbols from $\mathfrak{L}_C$ contained in $\mathfrak{L}_P$ are interpreted in the usual way, that is P does not redefine the predicates in $\mathfrak{L}_C$.

The subset A of B is *LP-definable* iff there exist a set of constants $C = \{c_1, \ldots, c_r\}$ and a pair $\langle P, F\rangle$, where P is a logic program and F is a new predicate symbol such that

$$s \in A \;\Leftrightarrow\; \exists\tau(\tau \in \mathfrak{T}_C \;\&\; \partial^C(\mathfrak{A}) \cup \{P\} \vdash F(\tau) \;\&\; \tau_{\mathfrak{A}} \simeq s),$$

where "$\vdash$" means deducibility in the sense of first order predicate calculus.

Let fix a finite part $\langle\alpha', H'\rangle$ such that $Dom(\alpha') = \{w_1, \ldots, w_r\}$ and $\alpha'(w_i) \simeq s_i$, $1 \le i \le r$. Let $c_{s_1}, \ldots, c_{s_r}$ be names for $s_1, \ldots, s_r$. We shall construct a logic program P such that the set LP-definable by $\langle P, F\rangle$ and $c_{s_1}, \ldots, c_{s_r}$ is sufficient for $\langle\alpha', H'\rangle$ and Γ. The program repeats the constructions in the proof of Theorem 1.

Let $\underline{0}$ and $\underline{nil}$ be new constant symbols, f_0 be a new unary functional symbol and h be a new binary functional symbol. Let $C = \{c_{s_1}, \ldots, c_{s_r}\}$, $\mathfrak{L}'_C = \{c_{s_1}, \ldots, c_{s_r}, \underline{0}, \underline{nil}, f_0, h, f_1, \ldots, f_n, T_0, \ldots, T_k\}$ and $\mathfrak{T}$ be the set of all terms without variables of $\mathfrak{L}'_C$. For each program P we consider Herbrand interpretations in $\mathfrak{L}_P$ with domain $\mathfrak{T}$. If Q is a predicate symbol of $\mathfrak{L}_P$ and I is a Herbrand interpretation of P, by $I(Q)$ we denote the corresponding predicate of $\mathfrak{T}$. An interpretation I of P is called a *model* for P iff all clauses of P are true in I.

For each natural n, by $\underline{n}$ we denote the term $f_0^n(\underline{0})$. Let $\underline{N}$ denote the set $\{\underline{n} \mid n \in N\}$.

It is well-known (see [3]) that:

Proposition 12. *For each r. e. subset W of N^k and for each k-ary predicate symbol Q there exists a logic program P with the following properties*:

(i) *If $(x_1, \ldots, x_k) \in W$, the $P \vdash Q(\underline{x}_1, \ldots, \underline{x}_k)$*;

(ii) *There exists a Herbrand interpretation I of P which is a model for P and*

$$I(Q)(a_1, \ldots, a_k) = 0$$
$$\Leftrightarrow\; \exists x_1 \ldots \exists x_k((x_1, \ldots, x_k) \in W \;\&\; a_1 = \underline{x}_1 \;\&\; \ldots a_k = \underline{x}_k).$$

Such an interpretation for P we shall call *standard*.

We define a *list* to be an element of $\mathfrak{T}$ such that: (i) $\underline{nil}$ is a list; (ii) if a is a term and b is a list, then $h(a, b)$ is a list.

We use the usual Prolog notation for lists.

Let $\underline{cod}$ be a new ternary predicate symbol and $\underline{nat}$ be a new unary predicate symbol. Let $P_{\underline{cod}}$ and $P_{\underline{nat}}$ be logic programs representing the r. e. sets $Cod = \{(\underline{x}, \underline{y}, \underline{z}) \mid x = \langle y, z\rangle\}$ and $Nat = \{\underline{x} \mid g(x) \notin Dom(\alpha')\}$ by $\underline{cod}$ and $\underline{nat}$, respectively, and

P_0

$\underline{tau}(\underline{w}_j, c_{s_j})$:- . $\quad j = 1, \ldots, r$

$\underline{tau}(X, f_i(V))$:- $\underline{cod}(X, \underline{i}, Y), \underline{tau}(Y, V)$. $\quad i = 1, \ldots n$

$P_{\underline{cod}}$.

The following proposition verifies the logic program P_0 using the method proposed in [3].

Proposition 13. *Let $x \in N$. Then $P_0 \vdash \underline{tau}(\underline{x}, \tau))$ iff $g(x) \in Dom(\alpha')$ and $\tau \equiv \tau^x$.*

Proof. The "if" part is easily proved by induction on $|x|$. To prove the "only if" part, let I be a standard Herbrand interpretation for $\underline{cod}$. We define I on τ as follows:

(i) $I(\underline{tau})(a, b) = 0$, if $a \notin \underline{N}$;

(ii) $I(\underline{tau})(a, b) = 0$, if $a \in \underline{N}$, $a = \underline{x}$, $g(x) \in Dom(\alpha')$ and $b \equiv \tau^x$;

(iii) $I(\underline{tau})(a, b) = 1$ otherwise.

It is easy to show that I is a model for P_0. □

Consider the following program:

P_1

$\underline{pi}([\,]) \text{ :- }.$

$\underline{pi}([X|Y]) \text{ :- } \underline{cod}(X, \underline{j}, Z), \underline{nat}(Z), \underline{pi}(Y). \quad j = 1, \ldots, n + k$

$\underline{pi}([X|Y]) \text{ :- } \underline{cod}(X, \underline{j}, Z), \underline{tau}(Z, V), T_0(f_j(V)), \underline{pi}(Y). \quad j = 1, \ldots, n$

$\underline{pi}([X|Y]) \text{ :- } \underline{cod}(X, \underline{j}, Z), \underline{tau}(Z, V), T_{j-n}(V), \underline{pi}(Y). \quad j = n + 1, \ldots, n + k$

$P_0.$

$P_{\underline{nat}}.$

Proposition 14. *Let $E = \{v_1, \ldots, v_l\}$ be a correct finite set. Then for each finite set G' of atoms without variables in $\mathfrak{L}'_C$ it is true that $P_1 \vdash G' \Rightarrow \underline{pi}([\underline{v}_1, \ldots, \underline{v}_l])$ iff $G' \supseteq \tilde{E}$.*

Proof. The "if" part is easily proved by induction on l. To prove the "only if" part, we define a class $\mathfrak{K}$ of Herbrand interpretations of P_1. The Herbrand interpretation I belongs to class $\mathfrak{K}$ iff:

(i) I is standard for $P_{\underline{nat}}$ and the predicate symbols in P_0 are interpreted as in the prove of the previous proposition;

(ii) Let $a \in \mathfrak{T}$. If a is not of the form $[\underline{v}_1, \ldots, \underline{v}_l]$ for any correct set $\{v_1, \ldots, v_l\}$, then $I(\underline{pi}(a)) \simeq 0$. If a is of the form $[\underline{v}_1, \ldots, \underline{v}_l]$ for some correct set $E = \{v_1, \ldots, v_l\}$, then $I(\underline{pi}(a)) \simeq 0$ iff there exists a finite set of atoms without variables $G = \{\beta_1, \ldots, \beta_q\}$, $q \geq 0$ of $\mathfrak{L}'_C$, such that $\tilde{E} \subseteq G$ and $I(\beta_j) \simeq 0$, $1 \leq j \leq q$.

It is easy to show that each I of $\mathfrak{K}$ is a model of P_1. Let $G = \{\beta_1, \ldots, \beta_q\}$ be a finite set of atoms without variables of $\mathfrak{L}'_C$, $E = \{v_1, \ldots, v_l\}$ be a correct set and $P_1 \vdash G' \Rightarrow \underline{pi}([\underline{v}_1, \ldots, \underline{v}_l])$. Let $I \in \mathfrak{K}$ be such that if β is an atom without variables of $\mathfrak{L}'_C$, then $I(\beta) \simeq 0 \Leftrightarrow \beta \in G$. Since I is a model of P_1, we have $I(\underline{pi})([\underline{v}_1, \ldots, \underline{v}_l]) \simeq 0$, that is $\tilde{E} \subseteq G$. □

Proposition 15. *For each enumeration operator Γ compatible with $\langle \alpha', H' \rangle$ there exists a logic program P such that the set A, LP-definable by $\langle P, F \rangle$ and $C = \{c_{s_1}, \ldots, c_{s_r}\}$, is sufficient for $\langle \alpha', H' \rangle$ and Γ.*

Proof. Let Γ be an arbitrary enumeration operator and W be the r. e. set which determines Γ, i. e. if R is a set of natural numbers, then $x \in \Gamma(R) \Leftrightarrow \exists u(\langle u, x\rangle \in W \;\&\; E_n \subseteq R)$. Let $W_1 = \{\langle u, x\rangle \mid \langle u, x\rangle \in W$ and E_u is correct$\}$. It is clear that W_1 is r. e. set. Let Q be a new predicate symbol and P_2 be the logic program representing W_1 by Q. Let $\underline{list}$ be a new binary predicate symbol and P_3 be a logic program such that:

(i) If u is a code of the finite set $\{v_1, \ldots, v_l\}$, $l \geq 0$, then $P_3 \vdash \underline{list}(u, [\underline{v}_1, \ldots, \underline{v}_l])$;

(ii) There exists a Herbrand interpretation I of P_3, which is a model of P_3, and if $E_u = \{v_1, \ldots, v_l\}$ for some natural u, then $I(\underline{list})(\underline{u}, b) = 0 \Leftrightarrow b = [\underline{v}_1, \ldots, \underline{v}_l]$).

Consider the following logic program:

P

$F(Y) \;\text{:-}\; Q(Z), \underline{cod}(Z, U, X), \underline{tau}(X, Y), T_0(Y), \underline{list}(U, V), \underline{pi}(V).$

$P_1.$

$P_2.$

$P_3.$

We shall use the next lemma, which proof is similar to the proof of the previous propositions.

Lemma 1. *Let G be a finite set of atoms without variables of $\mathfrak{L}'_C$. Then for each term τ without variables of $\mathfrak{L}'_C$, $P \vdash G \Rightarrow F(\tau)$ iff there exists $\langle u, x\rangle \in W_1$ such that:*

(i) *$g(x) \in Dom(\alpha')$ and $\tau \equiv \tau^x$;*

(ii) *$\tilde{E}_u \cup \{T_0(\tau)\} \subseteq G$.*

Let A be defined by $\langle P, F\rangle$ and $C = \{c_{s_1}, \ldots, c_{s_r}\}$. Let $\langle\alpha, \mathfrak{B}\rangle$ extend $\langle\alpha', H'\rangle$, $s \in \alpha(\Gamma(\langle\mathfrak{B}\rangle))$, x be such that $\alpha(x) \simeq s$ and let there exist $\langle u, x\rangle \in W_1$ such that $E_u \subseteq \mathfrak{B}$. This implies $(\tilde{E}_u)_{\mathfrak{A}} \simeq 0$. From the compatibility of Γ and $\langle\alpha', H'\rangle$ and the correctness of E_u it follows that $g(x) \in Dom(\alpha')$. And we also have $\tau^x_{\mathfrak{A}} \simeq \alpha(x) \simeq s$, hence $(T_0(\tau^x))_{\mathfrak{A}} \simeq true$. From the above arguments we obtain $P \vdash \tilde{E}_v \cup \{T_0(\tau^x)\} \Rightarrow F(\tau^x)$ and from the LP-definability of A it follows that $s \in A$, that is $\alpha(\Gamma(\langle\mathfrak{B}\rangle)) \subseteq A$.

Now let $s \in A$. Then there exists $\tau \in \mathfrak{T}_C$ such that $\tau_{\mathfrak{A}} \simeq s$ and $\partial(\mathfrak{A})^C \cup P \vdash H(\tau)$. From the reduction theorem it follows that there exists a finite set G of atoms without variables such that $G_{\mathfrak{A}} \simeq 0$ and $P \vdash G \Rightarrow F(\tau)$. Hence, there exists $\langle u, x\rangle \in W_1$ such that $g(x) \in Dom(\alpha')$, $\tau \equiv \tau^x$, $\tilde{E}_v \subseteq G$ and $\tau^x_{\mathfrak{A}} \simeq s$. Let

$$L = \{g(y) \mid \langle j, y\rangle \in E_v \;\&\; 1 \leq j \leq n + k \;\&\; g(y) \notin Dom(\alpha')\}.$$

Let $\alpha'' \equiv \alpha'$ and $H'' \simeq H' \cup L$. Let $\langle\alpha, \mathfrak{B}\rangle$ extend $\langle\alpha'', H''\rangle$. Then $\langle\alpha, \mathfrak{B}\rangle$ also extends $\langle\alpha', H'\rangle$. Consider the set E_v and let $t = \langle j, y\rangle \in E_v$. There are two possibilities:

1. $g(y) \in Dom(\alpha_q)$, hence $\Pi^t_{\mathfrak{A}} \simeq 0$, that is $\langle j, x\rangle \in \langle\mathfrak{B}\rangle$.

2. $g(y) \notin Dom(\alpha_q)$, hence $g(y) \in H''$. If $1 \leq j \leq n$, then $\langle j, x\rangle \in \langle\mathfrak{B}\rangle$ from the fact that $\langle\alpha, \mathfrak{B}\rangle$ is standard and $Dom(\alpha) \cap H'' = \varnothing$. In the case $n+1 \leq j \leq n+k$ we also obtain $\langle j, x\rangle \in \langle\mathfrak{B}\rangle$ from the definition of extension.

Finally, we have proved that $\tilde{E}_v \subseteq \langle\mathfrak{B}\rangle$, i.e. $x \in \Gamma(\langle\mathfrak{B}\rangle)$. On the other hand, $\alpha(x) \simeq \tau^x_{\mathfrak{A}} \simeq \tau_{\mathfrak{A}} \simeq s$ or $s \in \alpha(\Gamma(\langle\mathfrak{B}\rangle))$. Thus we obtained that A is sufficient for $\langle\alpha', H'\rangle$ and Γ. □

We already proved the next theorem.

Theorem 4. *Each weak-admissible set in $\mathfrak{A}$ is LP-definable.*

Theorem 5. *If the set A is LP-definable by $\langle P, H\rangle$ and $C = \{c_1, \dots, c_r\}$, then A is $\forall$-weak-admissible.*

Proof. Let $\mathfrak{L}_C = (c_1, \dots, c_r; f_1, \dots, f_n; T_0, \dots, T_k)$ be a first order language corresponding to $\mathfrak{A}$ and $\mathfrak{T}_C$ be the set of the terms without variables of $\mathfrak{L}_C$. Let

$$s \in A \iff \exists\tau(\tau \in \mathfrak{T}_C \;\&\; \partial^C(\mathfrak{A}) \cup P \vdash H(\tau) \;\&\; \tau_{\mathfrak{A}} \simeq s). \qquad (*)$$

Fix an arbitrary partial enumeration $\langle\alpha, \mathfrak{B}\rangle$ of $\mathfrak{A}$. We shall define a r. e. in $\mathfrak{B}$ set W of naturals such that $W \subseteq Dom(\alpha)$ and $\alpha(W) \equiv A$.

Let $\mathfrak{L} = (f_1, \dots, f_n; T_0, \dots, T_k)$ and let for each $(c_i)_{\mathfrak{A}} \in B$ choose $x_i \in N$ such that $\alpha(x_i) \simeq (c_i)_{\mathfrak{A}}$ (there exists such x_i since α is a surjective mapping). Let $K' = \{x_1, \dots, x_r\}$ and $K = \{\tilde{x} \mid x \in K'\}$, where $\tilde{x}$ is a new constant for each $x \in K'$ and $K \cap \mathfrak{L}_C = \varnothing$. Since K is a finite set, K is r. e. in $\mathfrak{B}$. Let $L_K = L \cup K$ and let $\mathfrak{B}^*$ be the enrichment of $\mathfrak{B}$ in $\mathfrak{L}_K$, where $\tilde{x}$ is interpreted as x. Consider the set $\mathfrak{T}_K$ of terms without variables of $\mathfrak{L}_K$. For each term τ of $\mathfrak{T}_K$ we define a term $\lceil\tau\rceil$ of $\mathfrak{T}_C$ by the following inductive clauses:

(i) if $\tau = \tilde{x}$ for some $\tilde{x} \in K$, then $\lceil\tau\rceil = c_i \Leftrightarrow \alpha(x) = (c_i)_{\mathfrak{A}}$;

(ii) if $\tau = f_i(\tau^1)$, then $\lceil\tau\rceil = f_i(\lceil\tau^1\rceil)$.

It is easily seen that for each term $\tau \in \mathfrak{T}_K$, $\tau_{\mathfrak{B}^*}$ is defined iff $\lceil\tau\rceil_{\mathfrak{A}}$ is defined and also that $\alpha(\tau_{\mathfrak{B}^*}) \simeq \lceil\tau\rceil_{\mathfrak{A}}$.

Let $\partial^C_1(\mathfrak{B}) = \{T_j(\tau) \mid 0 \leq j \leq k \;\&\; \tau \in \mathfrak{T}_K \;\&\; T_j(\lceil\tau\rceil) \in \partial^C(\mathfrak{A})\}$. The set $\partial^C_1(\mathfrak{B})$ is r. e. in $\mathfrak{B}$, because for $1 \leq j \leq k$ and $\tau \in \mathfrak{T}_K$ the following equivalences hold:

$$T_j(\tau) \in \partial^C_1(\mathfrak{B}) \iff T_j(\lceil\tau\rceil) \in \partial^C(\mathfrak{A}) \iff \Sigma_j(\lceil\tau\rceil_{\mathfrak{A}}) \simeq 0$$
$$\iff \Sigma_j(\alpha(\tau_{\mathfrak{B}^*})) \simeq 0 \iff \sigma_j(\tau_{\mathfrak{B}^*}) \simeq 0.$$

By changing each appearance of c_i to $\tilde{x}_i$ in P and τ we obtain $\tilde{P}$ and $\tilde{\tau}$. Now we define the set W by

$$x \in W \iff \exists\tilde{\tau}(\tilde{\tau} \in \mathfrak{T}_K \;\&\; \tilde{\tau}_{\mathfrak{B}^*} \simeq x \;\&\; \partial^C_1(\mathfrak{B}) \cup \{\tilde{P}\} \vdash H(\tilde{\tau})).$$

From that definition it is clear that W is r. e. in $\mathfrak{B}$. Since $Dom(\alpha)$ is closed with respect to φ_i, $1 \leq i \leq n$, we have $W \subseteq Dom(\alpha)$. And finally, from the constant theorem and reduction theorem it follows that $\alpha(W) = A$, which proves the theorem. □

Now we are ready to state the main results of the paper as corollaries of the theorems already proved.

Corollary 1. *The subset A of B is LP-definable iff there exists $\theta \in F(\mathfrak{A})$ such that $A = \theta(t_{01}, \ldots, t_{0r})$ for some fixed $(t_{01}, \ldots, t_{0r}) \in B^r$.*

Corollary 2. *The subset A of B is LP-definable iff A is weak-computable.*

7. CONCLUSIONS AND RELATED WORK

The subject of this paper is a semantics of logic programs without searching in the structure domain. The paper is a part of a more general exploration being performed at the Department of Mathematical Logic of Sofia University. All these works use the enumerations approach which is extremely suitable for problems of finding normal form of objects obtained by certain kind of computations. In [3] is considered a semantic, for which searching in the domain of the structure is allowed. There are also results for more general parameterized semantics.

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