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FINITE DEFORMATIONS OF TWO DROPS DUE TO ELECTRIC FIELD

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The finite deformations of two drops due to electric field are investigated in this article. The radii of the drops and the fluid phases could be different. Reynolds' number is assumed small enough to solve the problem in quasisteady Stokes' approximation. It is also supposed that the initial form of the drops is spherical and the fluids are homogenous, incompressible and Newtonian.

The electric and hydrodynamic problems are separated and the electric one has an influence on the hydrodynamic one through the boundary conditions. The Maxwell's equations are turned to Laplace's equations, and together with Stokes' equations they are solved by semianalytical-seminumerical method. We use boundary-integral type of these equations to solve them by the method of boundary elements. The kinematic condition gives a new form to the particles.

The results obtained indicate that interactions between two and three fluid phases, due to electric field, lead to deformations of the drops. The influence over the deformations of some dimensionless parameters of the problem has been given graphically.

Keywords: electric field, deformation, drop, boundary elements method, fluid phases, interfaces

Mathematics Subject Classification 2000: 76T30

1. INTRODUCTION

Basic ideas for investigating the matter of fluid particles deformations have been presented first by G. I. Taylor (1932). In his next paper (1934) Taylor has found the critical velocity of shear flow, after which a drop set in the flow starts to elongate. In [26] it has been proved that in uniform flow in Stokes approximation an initially spherical particle remains spherical without any deformations.

E. Chervenivanova and Z. Zapryanov obtain small deformations of drop moving with a uniform velocity in spherical container, full with viscous fluid. Although the flow is uniform, there are deformations of the drop, because it is in a container which causes the deformations.

Uijttewaal et al. (1993) solve numerically the three-dimensional problem of drop in linear shear flow moving to a plane wall, using the boundary element method.

The problems of single drop subjected in viscous flow are in the basis for solving problems of compound drops (drop in drop), drop near a plane wall or two separated drops.

The technique "method of reflection", which is used for the first time by Smoluchowski (1911), is in the base of the first systematic investigations of the dynamics of two fluid drops made by Happel & Brenner (1965).

Small deformations of two fluid drops have been presented first in [3]. Deformations of two fluid droplets, drop and bubble, and drop and rigid particle in uniform flow are obtained. A parametric analysis of the small deformations relative to the distance between drops and the ration of viscosities of the different phases is made. "Dimple" formation is one of the basic results of the paper.

The influence of electric field on a water drop has been investigated experimentally in [29] and [12]. The authors have found the critical value of dimensionless parameter (E^*) after which the drop breaks up. Taylor (1964) improves theoretically this value supposing that the drop preserves its spherical form until the break up. In [1, 5, 9, 25] a drop's break up with conical tips is examined. Ramos & Castellanos (1994) present theoretical result for the influence of the coefficients of permittivity and conductivity on the conical tips formation. Torza, Cox & Mason (1971) have found experimentally another model of breaking up a drop, which is divided into two spherical parts connected with a thin "throat". Sherwood (1988), using the method of boundary elements, solves numerically the problem of a single fluid particle deformation under the influence of electric field.

The form which two equal fluid drops achieve in the presence of electric field is given experimentally by O'Konski & Thacker (1953). In the papers [5, 13, 23] the authors show that due to the same electric field but with different parameters of the fluid phases (conductivities, permittivities) there are deformations of the interfaces based on electrostatic charge. In [2, 11, 24] a couple of equal water drops situated in an electric field is investigated experimentally. Sozou (1975), using bipolar coordinate system, presents semianalytical decision for velocities in and out of the drops, presuming keeping the spherical form.

2. FORMULATION OF THE PROBLEM

The problem for defining the finite deformations of two fluid drops due to the electric field is separated into two problems — electrostatic and hydrodynamic. The Navier-Stokes equations and the Maxwell's equations are describing most precisely that problem. In low Reynolds number and quasisteady approximation they turn respectively into Stokes equations for velocities and Laplace's equations for electric potentials, as written below.

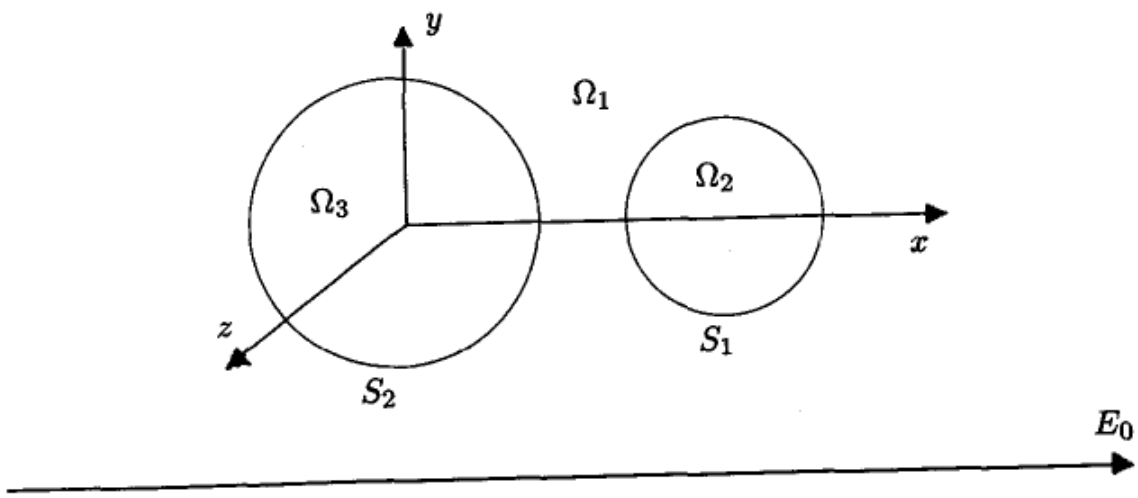

Fig. 1. Scheme of two drops in the presence of electric field

The drops on Fig. 1 are compounded of fluid 2 with viscosity μ_2, conductivity σ_2, permittivity ε_2, and fluid 3 with viscosity μ_3, conductivity σ_3, permittivity ε_3. The electric field that acts on the axis connecting the centres of the drops is with intensity E_0. Under its influence the interfaces of the drops deform. The initial form of fluid drops is spherical with undistorted radius R_1 of the first sphere and undistorted radius R_2 of the second one. With S_1 is marked the interface between phase 1 and phase 2, and with S_2 — the interface between phase 1 and phase 3. The interfacial tensions over S_1 and S_2 are γ_1 and γ_2, respectively. The fluids 1, 2 and 3 are situated in Ω_1, Ω_2 and Ω_3, respectively, while Ω_1 is the infinite area outside the drop (Fig. 1).

At each point the electric potential and the velocity of the flow at each moment is governed by the following equations:

— Laplace's equations: $\Delta\varphi^k = 0 \quad (k = 1, 2, 3);$ (2.1)

— discontinuity equations: $\dfrac{\partial u_i^k}{\partial x_i} = 0 \quad (i, k = 1, 2, 3);$ (2.2)

— Stokes' equations: $\dfrac{\partial \sigma_{ij}^k}{\partial x_j} = 0 \quad (i, j, k = 1, 2, 3)$ (2.3)

where σ_{ij}^k is the stress tensor $\sigma_{ij}^k = -p^k\delta_{ij} + \mu_k\left(\dfrac{\partial u_i^k}{\partial x_j} + \dfrac{\partial u_j^k}{\partial x_i}\right)$.

The index $k = 1$ for $\boldsymbol{x} \in \Omega_1$, $k = 2$ for $\boldsymbol{x} \in \Omega_2$ and $k = 3$ for $\boldsymbol{x} \in \Omega_3$, while p^k is the hydrodynamic pressure of the respective fluid. The electric potential in the three phases satisfies the following boundary conditions:

$$\varphi^1(\boldsymbol{x}_0) \to E_0 x_0^1, \qquad |\boldsymbol{x}_0| \to \infty, \tag{2.1.a}$$
$$\varphi^1(\boldsymbol{x}_0) = \varphi^2(\boldsymbol{x}_0), \qquad \boldsymbol{x}_0 \in S_1, \tag{2.1.b}$$
$$\varphi^1(\boldsymbol{x}_0) = \varphi^3(\boldsymbol{x}_0), \qquad \boldsymbol{x}_0 \in S_2, \tag{2.1.c}$$

$$\sigma_1 \frac{\partial \varphi^1}{\partial n}(x_0) = \sigma_2 \frac{\partial \varphi^2}{\partial n}(x_0), \qquad x_0 \in S_1, \tag{2.1.d}$$

$$\sigma_1 \frac{\partial \varphi^1}{\partial n}(x_0) = \sigma_3 \frac{\partial \varphi^3}{\partial n}(x_0), \qquad x_0 \in S_2, \tag{2.1.e}$$

where E_0 is the intensity of the electric field, x_0^1 is the x-component in Decart coordinate system $Oxyz$ of the vector x_0, and σ_1, σ_2, σ_3 are the electric conductivities of the respective fluids, and $\dfrac{\partial}{\partial n}$ is the normal derivative to the surface, pointed out of the respective domain.

The flow field is governed by the following boundary conditions:

$$u_i^1(x_0) \to 0, \qquad |x_0| \to \infty, \tag{2.3.a}$$

$$u_i^1(x_0) = u_i^2(x_0), \qquad x_0 \in S_1, \tag{2.3.b}$$

$$\sigma_{ij}^1(x_0)n_j(x_0) - \sigma_{ij}^2(x_0)n_j(x_0)$$
$$= \gamma_1 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^1(x_0)n_j(x_0) - \tau_{ij}^2(x_0)n_j(x_0)\right), \qquad x_0 \in S_1, \tag{2.3.c}$$

$$u_i^1(x_0) = u_i^3(x_0), \qquad x_0 \in S_2, \tag{2.3.d}$$

$$\sigma_{ij}^1(x_0)n_j(x_0) - \sigma_{ij}^3(x_0)n_j(x_0)$$
$$= \gamma_2 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^1(x_0)n_j(x_0) - \tau_{ij}^3(x_0)n_j(x_0)\right), \qquad x_0 \in S_2. \tag{2.3.e}$$

Here n is the single outer normal to the interface S_1 or S_2,

$$\tau_{ij}^k = -\frac{\varepsilon_k}{4\pi}\left(\frac{(\mathbf{E}^k)^2}{2}\delta_{ij} - \mathbf{E}_i^k \mathbf{E}_j^k\right)$$

is the Maxwell's electric stress tensor for the respective phases ($k = 1, 2, 3$), where ε_k is the electric permittivity of the different phases and $\mathbf{E}^k = -\nabla\varphi^k$. Let us assume that S_1 and S_2 are Lyapunov's surfaces. The solution of (2.1) with boundary conditions (2.1.a–e) gives us the electric potentials at each moment and at every point of the three phases. The solution of (2.2), (2.3) with boundary conditions (2.3.a–e) gives us the velocity at each moment and at every point of S_1 and S_2. The deformation of the interfaces is determined at each moment by the normal component of the velocity and the kinematic condition:

$$\frac{dx_s}{dt} = n_i(u_i \cdot n_i) = n_i \cdot u_n. \tag{2.4}$$

Here x_s is a point of the respective surface S_1 or S_2, while u_n is the normal component of the velocity at this point.

Following Greengard & Moura (1994), the integral equations, which determine the potentials of the electric field on the interfaces, are solutions of the system (2.1) with boundary conditions (2.1.a–e) in the single-layer integral form:

$$\varphi(x_0) = \psi(x_0) + \int_S G(x_0, x)\rho(x)\, dS_x, \tag{2.5}$$

where $\varphi(\boldsymbol{x}_0)$ is the total potential of the field at the point $\boldsymbol{x}_0$:

$$\varphi^1(\boldsymbol{x}_0) = \varphi(\boldsymbol{x}_0)_{|\Omega_1}, \qquad \varphi^2(\boldsymbol{x}_0) = \varphi(\boldsymbol{x}_0)_{|\Omega_2},$$
$$\varphi^3(\boldsymbol{x}_0) = \varphi(\boldsymbol{x}_0)_{|\Omega_3}, \qquad \psi(\boldsymbol{x}_0) = E_0.x_0^1,$$

$\rho(\boldsymbol{x})$ is an unknown function of distribution, $G(\boldsymbol{x}_0, \boldsymbol{x})$ is a Green's function for the domain $S = S_1 \cup S_2$, which for our case is $G(\boldsymbol{x}_0, \boldsymbol{x}) = \dfrac{1}{|\boldsymbol{x} - \boldsymbol{x}_0|}$.

By substituting (2.1.d–e) in (2.5) and using the single-layer potential theory we derive:

$$\sigma_k \left[\frac{\partial \psi(\boldsymbol{x}_0)}{\partial \boldsymbol{n}} - \frac{1}{2}\rho(\boldsymbol{x}_0) + \int_S \frac{\partial G}{\partial \boldsymbol{n}}(\boldsymbol{x}_0, \boldsymbol{x}) \rho(\boldsymbol{x})\, dS_x \right]$$
$$= \sigma_1 \left[\frac{\partial \psi(\boldsymbol{x}_0)}{\partial \boldsymbol{n}} + \frac{1}{2}\rho(\boldsymbol{x}_0) + \int_S \frac{\partial G}{\partial \boldsymbol{n}}(\boldsymbol{x}_0, \boldsymbol{x}) \rho(\boldsymbol{x})\, dS_x \right], \tag{2.6}$$
$$\rho(\boldsymbol{x}_0) - 2\lambda_k \int_S \frac{\partial G}{\partial \boldsymbol{n}}(\boldsymbol{x}_0, \boldsymbol{x}) \rho(\boldsymbol{x})\, dS_x = \lambda_k \frac{\partial \psi(\boldsymbol{x}_0)}{\partial \boldsymbol{n}},$$

where $\lambda_k = \dfrac{\sigma_k - \sigma_1}{\sigma_k + \sigma_1}$, $k = 2, 3$.

In order to solve the hydrodynamic problem (2.2), (2.3) with boundary conditions (2.3.a–e), following Power [14], we use Green's integral representation formulae for Stokes equations to get to integral equations, which determine the velocities on the interfaces:

$$\begin{aligned}\frac{1 + (\mu_1/\mu_2)}{2} \boldsymbol{u}_i^1(\boldsymbol{x}_0) = & -\left(1 - \frac{\mu_1}{\mu_2}\right) \int_{S_1} T_{ijk}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}_j^1(\boldsymbol{x}) \boldsymbol{n}_k(\boldsymbol{x})\, dS_x \\ & -\left(1 - \frac{\mu_1}{\mu_3}\right) \int_{S_2} T_{ijk}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}_j^3(\boldsymbol{x}) \boldsymbol{n}_k(\boldsymbol{x})\, dS_x \\ & -\frac{1}{\mu_1} \int_{S_1} J_{ij}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_1 \boldsymbol{n}_j \frac{\partial \boldsymbol{n}_k}{\partial \boldsymbol{x}_k} - \left(\tau_{jk}^1 \boldsymbol{n}_k - \tau_{jk}^2 \boldsymbol{n}_k\right) \right) dS_x \\ & -\frac{1}{\mu_1} \int_{S_2} J_{ij}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_2 \boldsymbol{n}_j \frac{\partial \boldsymbol{n}_k}{\partial \boldsymbol{x}_k} - \left(\tau_{jk}^1 \boldsymbol{n}_k - \tau_{jk}^3 \boldsymbol{n}_k\right) \right) dS_x\end{aligned} \tag{2.7}$$

for each $\boldsymbol{x}_0 \in S_1$,

$$\begin{aligned}\frac{1 + (\mu_1/\mu_3)}{2} \boldsymbol{u}_i^3(\boldsymbol{x}_0) = & -\left(1 - \frac{\mu_1}{\mu_2}\right) \int_{S_1} T_{ijk}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}_j^1(\boldsymbol{x}) \boldsymbol{n}_k(\boldsymbol{x})\, dS_x \\ & -\left(1 - \frac{\mu_1}{\mu_3}\right) \int_{S_2} T_{ijk}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}_j^3(\boldsymbol{x}) \boldsymbol{n}_k(\boldsymbol{x})\, dS_x\end{aligned} \tag{2.8}$$

$$- \frac{1}{\mu_1} \int\limits_{S_2} J_{ij}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_2 n_j \frac{\partial n_k}{\partial x_k} - \left(\tau^1_{jk} n_k - \tau^3_{jk} n_k \right) \right) dS_x$$

$$- \frac{1}{\mu_1} \int\limits_{S_1} J_{ij}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_1 n_j \frac{\partial n_k}{\partial x_k} - \left(\tau^1_{jk} n_k - \tau^2_{jk} n_k \right) \right) dS_x$$

for each $\boldsymbol{x}_0 \in S_2$.

The equations (2.5)–(2.8) are dimensionlized to a form that is given in the next part and the following dimensionless parameters are included:

$E_\gamma = \dfrac{\varepsilon_1 E_0^2 R_1}{\gamma_2}$ — dimensionless parameter that indicates the relation between the electric and the capillary forces;

$\mu_{12} = \dfrac{\mu_1}{\mu_2}$, $\mu_{13} = \dfrac{\mu_1}{\mu_3}$ — the relation of the viscosities of the different neighbouring phases;

$\varepsilon_{21} = \dfrac{\varepsilon_2}{\varepsilon_1}$, $\varepsilon_{31} = \dfrac{\varepsilon_3}{\varepsilon_1}$ — the relation of the electric permittivity of the different neighbouring phases;

$\sigma_{12} = \dfrac{\sigma_1}{\sigma_2}$, $\sigma_{13} = \dfrac{\sigma_1}{\sigma_3}$ — the relation of the electric conductivity of the different neighbouring phases;

$\gamma_{12} = \dfrac{\gamma_1}{\gamma_2}$ — the relation of the interface tension coefficient of the two surfaces of the drops;

$R_{12} = \dfrac{R_1}{R_2}$ — the relation between the radii of the two drops.

To accomplish the formulation of the problem, we should say that on each time step we solve first the electrostatic problem, which has an influence on the hydrodynamic one, by Maxwell's electric stress tensor. On its turn, the solving of the hydrodynamic problem gives us the velocities of the fluids in the different phases. Using the kinematic condition for the normal velocity components on the fluid surfaces, we get their deformation. With the new form (changed boundary conditions) we solve once again the electrostatic problem and after that the hydrodynamic one, since the number of time steps determines how many times this procedure will be used. The criteria for ending the procedure are reaching an equilibrium form of the drops or "break up".

3. ALGORITHM FOR DETERMINING THE DEFORMATIONS OF TWO DROPS DUE TO ELECTRIC FIELD

The main steps of the algorithm followed are:

- change of the co-ordinate system from Decart's to cylindrical, in order to transform the boundary integrals to one-dimensional;
- introduction of boundary elements over the boundaries of the domains — arcs of circles;

- introduction of local polar co-ordinate system for each boundary element;
- calculation of the integrals of the single- and double-layer over each boundary element;
- subtraction of the integrals singularities;
- calculation of the velocity on the interfaces;
- determination of the drops form from the kinematic condition.

Due to the axisymmetric flow of the problem, we change the co-ordinate system to a cylindrical one (x, σ, ϕ), in which none of the unknown functions depends on the azimuthal angle ϕ: $\varphi_\phi = \rho_\phi = u_\phi = n_\phi = 0$ (Fig. 2).

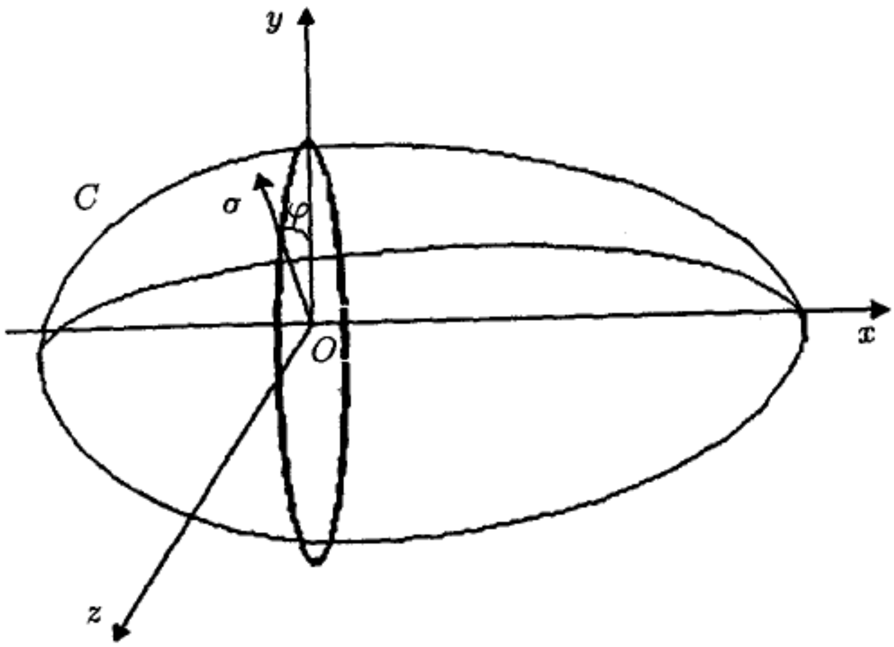

Fig. 2. Scheme of cylindrical co-ordinate system for axisymmetric flow

The normal $\boldsymbol{n}$, the velocity $\boldsymbol{u}$, the electric potential φ and the unknown function of distribution ρ are presented through the new co-ordinates:

$$\boldsymbol{x} = [x, \sigma\cos\phi, \sigma\sin\phi], \quad \boldsymbol{x}_0 = [x_0, \sigma_0\cos\phi_0, \sigma_0\sin\phi_0], \quad r_x = x_0 - x, \quad \boldsymbol{r}_x = \boldsymbol{x}_0 - \boldsymbol{x}$$
$$[\boldsymbol{n}_x, \boldsymbol{n}_y, \boldsymbol{n}_z] = [\boldsymbol{n}_x, \boldsymbol{n}_\sigma\cos\phi, \boldsymbol{n}_\sigma\sin\phi], \quad [\boldsymbol{u}_x, \boldsymbol{u}_y, \boldsymbol{u}_z] = [\boldsymbol{u}_x, \boldsymbol{u}_\sigma\cos\phi, \boldsymbol{u}_\sigma\sin\phi],$$
$$[\varphi_x, \varphi_y, \varphi_z] = [\varphi_x, \varphi_\sigma\cos\phi, \varphi_\sigma\sin\phi], \quad [\rho_x, \rho_y, \rho_z] = [\rho_x, \rho_\sigma\cos\phi, \rho_\sigma\sin\phi].$$

The differential dS is presented through the formula $dS = \sigma\, d\phi dl$, where dl is the elementary length of the curve $C = C_1 \cup C_2$ projection of $S = S_1 \cup S_2$ in the meridian plain Oxy, i.e. C_1 is the projection of S_1, while C_2 is the projection of S_2. Thus, we get to the following equations:

$$\varphi(\boldsymbol{x}_0) = \psi(\boldsymbol{x}_0) + \frac{1}{2\pi}\int\limits_C \rho(l) K^S(\boldsymbol{x}_0, l)\sigma(l)\, dl(\boldsymbol{x}), \tag{3.1}$$

$$\rho(\boldsymbol{x}_0) - \frac{\lambda_k}{\pi}\int\limits_C \rho(l) K^D(\boldsymbol{x}_0, l)\sigma(l)\, dl(\boldsymbol{x}) = \lambda_k \frac{\partial\psi(\boldsymbol{x}_0)}{\partial\boldsymbol{n}}, \tag{3.2}$$

where

$$K^S(\boldsymbol{x}_0, l) = \int_0^{2\pi} \frac{d\phi(\boldsymbol{x})}{|\boldsymbol{x}_0 - \boldsymbol{x}|} = \int_0^{2\pi} \frac{d\phi(\boldsymbol{x})}{|\boldsymbol{r}_x|} = I_{10}(\boldsymbol{x}_0, \sigma, \sigma_0),$$

$$K^D(\boldsymbol{x}_0, l) = \int_0^{2\pi} \frac{(\boldsymbol{x}_0 - \boldsymbol{x}) \cdot \boldsymbol{n}(\boldsymbol{x})}{|\boldsymbol{x}_0 - \boldsymbol{x}|^3} \, d\phi(\boldsymbol{x}) = \int_0^{2\pi} \frac{\boldsymbol{r}_x \cdot \boldsymbol{n}(\boldsymbol{x})}{|\boldsymbol{r}_x|^3} \, d\phi(\boldsymbol{x})$$
$$= (r_x \boldsymbol{n}_x + \sigma_0 \boldsymbol{n}_\sigma) I_{30}(\boldsymbol{x}_0, \sigma, \sigma_0) - \sigma_0 \boldsymbol{n}_\sigma I_{31}(\boldsymbol{x}_0, \sigma, \sigma_0).$$

$I_{mn}(x_0, \sigma, \sigma_0)$ are functions defined by:

$$I_{mn} = \int_0^{2\pi} \frac{\cos^n \phi}{\left(r_x^2 + \sigma^2 + \sigma_0^2 - 2\sigma\sigma_0 \cos\phi\right)^{m/2}} \, d\phi$$
$$= \frac{4k'^m}{(4\sigma\sigma_0)^{m/2}} \int_0^{\pi/2} \frac{\left(2\cos^2\omega - 1\right)^n}{\left(1 - k'^2 \cos^2\omega\right)^{m/2}} \, d\omega,$$

where $k'^2 = \dfrac{4\sigma\sigma_0}{r_x^2 + (\sigma + \sigma_0)^2}$.

$$\frac{1}{2}(\mu_{12} + 1)\boldsymbol{u}^1_\alpha(\boldsymbol{x}_0) + (1 - \mu_{12}) \int_{C_1} q_{\alpha\beta\gamma}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}^1_\beta(\boldsymbol{x}) \boldsymbol{n}_\gamma(\boldsymbol{x}) \, dl(\boldsymbol{x}) \tag{3.3}$$
$$+ (1 - \mu_{13}) \int_{C_2} q_{\alpha\beta\gamma}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}^3_\beta(\boldsymbol{x}) \boldsymbol{n}_\gamma(\boldsymbol{x}) \, dl(\boldsymbol{x})$$
$$= -\int_{C_1} M_{\alpha\beta}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_{12} \boldsymbol{n}_\beta \nabla \cdot \boldsymbol{n} - E_\gamma \left(\tau^1_{\alpha\beta} \boldsymbol{n}_\beta - \tau^2_{\alpha\beta} \boldsymbol{n}_\beta \varepsilon_{21}\right)\right) dl(\boldsymbol{x})$$
$$- \int_{C_2} M_{\alpha\beta}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\boldsymbol{n}_\beta \nabla \cdot \boldsymbol{n} - E_\gamma \left(\tau^1_{\alpha\beta} \boldsymbol{n}_\beta - \tau^3_{\alpha\beta} \boldsymbol{n}_\beta \varepsilon_{31}\right)\right) dl(\boldsymbol{x}),$$

$$\frac{1}{2}(1 + \mu_{13})\boldsymbol{u}^3_\alpha(\boldsymbol{x}_0) + (1 - \mu_{12}) \int_{C_1} q_{\alpha\beta\gamma}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}^1_\beta(\boldsymbol{x}) \boldsymbol{n}_\gamma(\boldsymbol{x}) \, dl(\boldsymbol{x}) \tag{3.4}$$
$$+ (1 - \mu_{13}) \int_{C_2} q_{\alpha\beta\gamma}(\boldsymbol{x}_0, \boldsymbol{x}) \boldsymbol{u}^3_\beta(\boldsymbol{x}) \boldsymbol{n}_\gamma(\boldsymbol{x}) \, dl(\boldsymbol{x})$$
$$= -\int_{C_1} M_{\alpha\beta}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\gamma_{12} \boldsymbol{n}_\beta \nabla \cdot \boldsymbol{n} - E_\gamma \left(\tau^1_{\alpha\beta} \boldsymbol{n}_\beta - \tau^2_{\alpha\beta} \boldsymbol{n}_\beta \varepsilon_{21}\right)\right) dl(\boldsymbol{x})$$
$$- \int_{C_2} M_{\alpha\beta}(\boldsymbol{x}_0, \boldsymbol{x}) \left(\boldsymbol{n}_\beta \nabla \cdot \boldsymbol{n} - E_\gamma \left(\tau^1_{\alpha\beta} \boldsymbol{n}_\beta - \tau^3_{\alpha\beta} \boldsymbol{n}_\beta \varepsilon_{31}\right)\right) dl(\boldsymbol{x}).$$

The indices α, β, γ are for x or σ, and we denote by them the axial and radial components, respectively.

The matrices M and q are defined as in [30]:

$$M(\boldsymbol{x}_0, \boldsymbol{x}) = \begin{bmatrix} M_{xx} & M_{x\sigma} \\ M_{\sigma x} & M_{\sigma\sigma} \end{bmatrix} = \sigma \int_0^{2\pi} \begin{bmatrix} J_{xx} & J_{xy}\cos\phi + J_{xz}\sin\phi \\ J_{xy} & J_{yy}\sin\phi + J_{yz}\cos\phi \end{bmatrix} d\phi,$$

$$\begin{bmatrix} q_{xxx} & q_{xx\sigma} \\ q_{x\sigma x} & q_{x\sigma\sigma} \end{bmatrix} (\boldsymbol{x}_0, \boldsymbol{x})$$

$$= \sigma \int_0^{2\pi} \begin{bmatrix} T_{xxx} & T_{xxy}\cos\phi + T_{xxz}\sin\phi \\ T_{xxy}T_{xzz} + T_{xxz}\sin\phi & T_{xyy}\cos^2\phi + T_{xzz}\sin^2\phi + 2T_{xyz}\sin\phi\cos\phi \end{bmatrix} d\phi,$$

$$\begin{bmatrix} q_{\sigma xx} & q_{\sigma x\sigma} \\ q_{\sigma\sigma x} & q_{\sigma\sigma\sigma} \end{bmatrix} (\boldsymbol{x}_0, \boldsymbol{x})$$

$$= \sigma \int_0^{2\pi} \begin{bmatrix} T_{xxy} & T_{xyy}\cos\phi + T_{xyz}\sin\phi \\ T_{xyy}T_{xzz} + T_{xyz}\sin\phi & T_{yyy}\cos^2\phi + T_{yzz}\sin^2\phi + 2T_{yyz}\sin\phi\cos\phi \end{bmatrix} d\phi.$$

Formulated by the integrals $I_{mn}(x_0, \sigma, \sigma_0)$, we have

$$M = \sigma \begin{bmatrix} I_{10} + r_x^2 I_{30} & -r_x(\sigma I_{30} - \sigma_0 I_{31}) \\ -r_x(\sigma I_{31} - \sigma_0 I_{30}) & I_{11} + \left(\sigma^2 + \sigma_0^2\right) I_{31} - \sigma\sigma_0(I_{30} + I_{32}) \end{bmatrix},$$

$$\begin{bmatrix} q_{xxx} \\ q_{xx\sigma} = q_{x\sigma x} \\ q_{x\sigma\sigma} \end{bmatrix} = 6\sigma r_x \begin{bmatrix} -r_x^2 I_{50} \\ r_x(\sigma I_{50} - \sigma_0 I_{51}) \\ -\left(\sigma^2 I_{50} + \sigma_0^2 I_{52} - 2\sigma\sigma_0 I_{51}\right) \end{bmatrix},$$

$$\begin{bmatrix} q_{\sigma xx} \\ q_{\sigma x\sigma} = q_{\sigma\sigma x} \\ q_{\sigma\sigma\sigma} \end{bmatrix} = 6\sigma \begin{bmatrix} r_x^2(\sigma I_{51} - \sigma_0 I_{50}) \\ -r_x \left[\left(\sigma^2 + \sigma_0\right)^2 I_{51} - \sigma\sigma_0(I_{50} + I_{52})\right] \\ \sigma^3 I_{51} - \sigma^2\sigma_0(I_{50} + 2I_{52}) + \sigma\sigma_0^2(I_{53} + 2I_{51}) - \sigma_0^3 I_{52} \end{bmatrix}.$$

The integrals I_{mn} could be expressed by elliptic integrals of first and second kind — $F(k')$ and $E(k')$, which are calculated numerically:

$$F(k') = \int_0^{\pi/2} \frac{d\omega}{\left(1 - k'^2\cos^2\omega\right)^{1/2}}, \qquad E(k') = \int_0^{\pi/2} \left(1 - k'^2\cos^2\omega\right)^{1/2} d\omega.$$

The integrals of single and double layer integrals in (3.1)–(3.4) have singularities, and in the singularity point these integrals are calculated using the formulae derived by Pozrikidis [15].

The system (3.1)–(3.4) is solved using the method of boundary elements, and the algebraic system following the electric problem (3.1), (3.2) is solved by Gauss' elimination, while the one following the hydrodynamic part (3.3), (3.4) is solved by the iterative method. For the calculations a project in Code Warrior C has been conducted, as the main results have been obtained through Power Mac 200/6400 in the Laboratory of the Department of Mechanics of Continua at the Faculty of Mathematics and Informatics at the Sofia University "St. Kl. Ohridski".

For the determination of the drops form on each time-step, we use the kinematic condition of the following type:

$$x_s^{\text{new}} = x_s + n_i(u_i \cdot n_i)dt, \qquad \text{where } dt \text{ is a preliminary set time-step.}$$

We assume that the form reaches the equilibrium when the normal component of the velocity becomes less than the preliminary set minimum at every point of the interfaces. Another criterion for the end of the procedure is the normal component to become bigger than the initially set number; then we consider the drop's break-up.

4. RESULTS

The algorithm for obtaining the finite deformations of two drops due to electric field has been tested for a single drop in the presence of an electric field and it has shown a good agreement with the results of Sherwood [18].

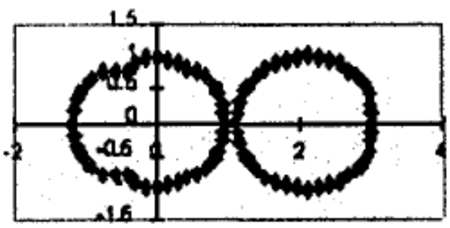

Fig. 3

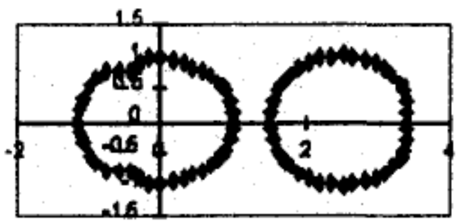

Fig. 4

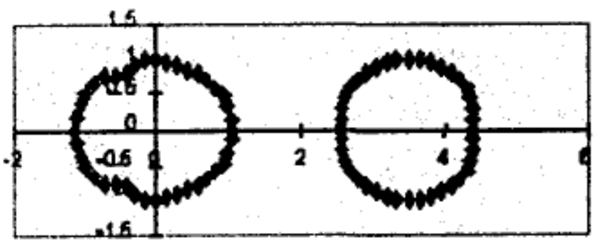

Fig. 5

The deformation of fluid interfaces when they are with equal radii does not depend essentially on the distance between the drops in low intensities $E_\gamma = 0.4$ of the electric field. The initial distance between the centres of the drops on Fig. 3 is 2.1 R1, on Fig. 4 is 2.5 R1, on Fig. 5 is 3.5 R1 with $\mu_{12} = 0.50$, $\mu_{13} = 1.50$, $E_\gamma = 0.4$, $\varepsilon_{21} = 2.0$, $R_{12} = 1.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$.

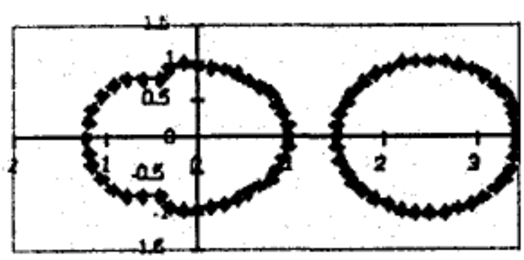

Fig. 6

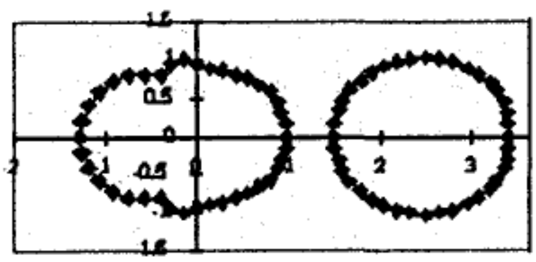

Fig. 7

When we increase the intensity of the electric field, the form reached by the first drop is elongating, till the second one retains almost spherical. On Fig. 6 $E_\gamma = 1.0$, on Fig. 7 $E_\gamma = 2.0$, on Fig. 8 $E_\gamma = 5.0$ with $\mu_{12} = 0.50$, $\varepsilon_{21} = 2.0$, $\mu_{13} = 1.5$, $R_{12} = 1.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$.

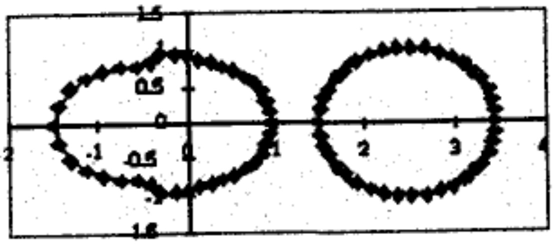

Fig. 8

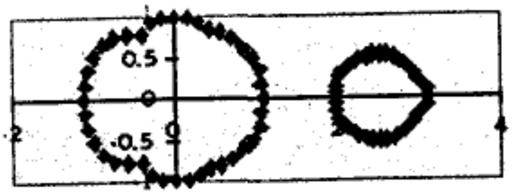

Fig. 9

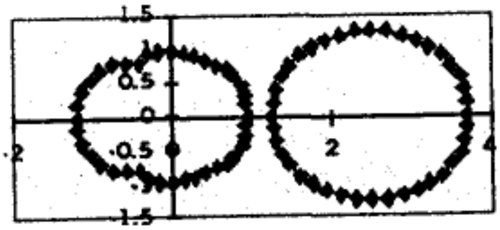

Fig. 10

The ratio between the radii of the two drops causes different pictures of deformation as shown on Fig. 9–11. On Fig. 9 $R_{12} = 0.5$, on Fig. 10 $R_{12} = 1.25$, on Fig. 11 $R_{12} = 1.49$ with $\mu_{12} = 0.5$, $E_\gamma = 0.5$, $\mu_{13} = 2.0$, $\varepsilon_{21} = 2.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$. The change of the ratio causes increase of the influence of the initially bigger drop to the smaller one.

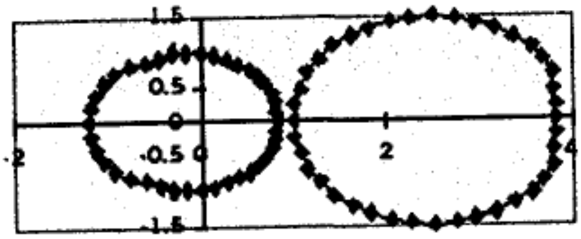

Fig. 11

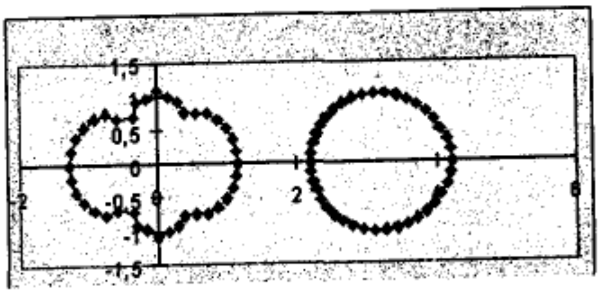

Fig. 12

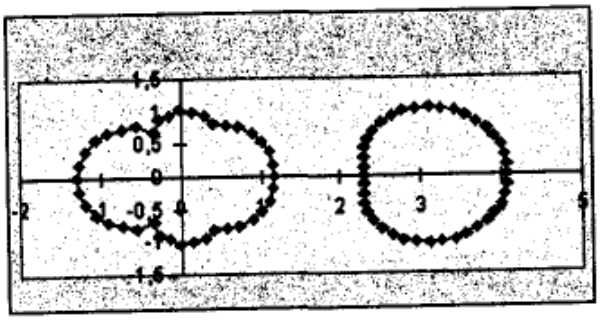

Fig. 13

On Fig. 12–14 the ratio between the viscosities is changed from $\mu_{13} = 1.0$ on Fig. 12, $\mu_{13} = 2.0$ on Fig. 13 and $\mu_{13} = 4.0$ on Fig. 14 with $R_{12} = 1.0$, $E_\gamma = 1.8$, $\mu_{12} = 2.0$, $\varepsilon_{21} = 2.0$, $\varepsilon_{31} = 1.5$, $\sigma_{12} = 20.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$. That shows that a change of viscosity between two phases causes a change of deformations on the both drops.

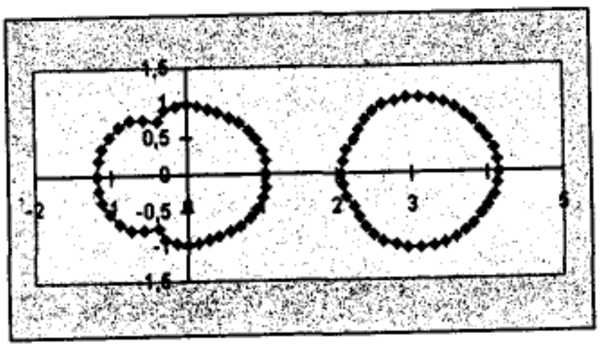

Fig. 14

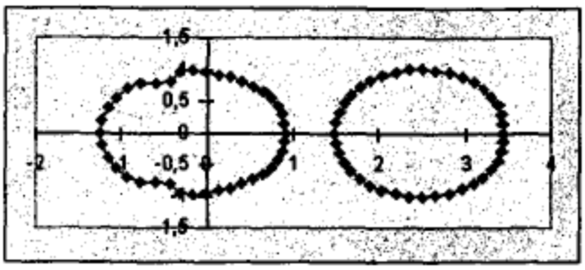

Fig. 15

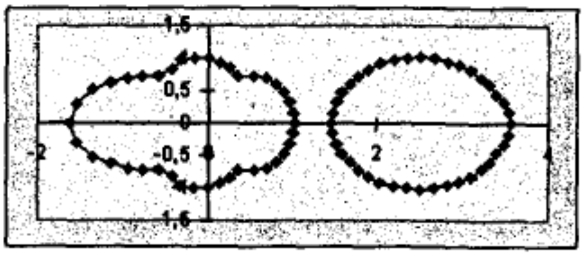

Fig. 16

On Fig. 15 and 16 the permittivity ratio ε_{31} changes from 2.0 to 10.0 on Fig. 16 with $R_{12} = 1.0$, $E_\gamma = 1.8$, $\mu_{13} = 1.5$, $\mu_{12} = 0.5$, $\varepsilon_{21} = 5.0$, $\sigma_{12} = 20.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$. That indicates that the increase of permittivity ratio between the phases 1 and 2 tends the drop to elongate in the direction opposite to the neighbour drop, but the configuration of Fig. 16 ($\varepsilon_{31} = 10.0$) is not stable and after some time steps a "break-up" appears.

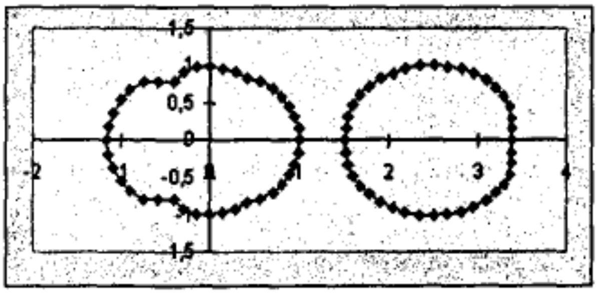

Fig. 17

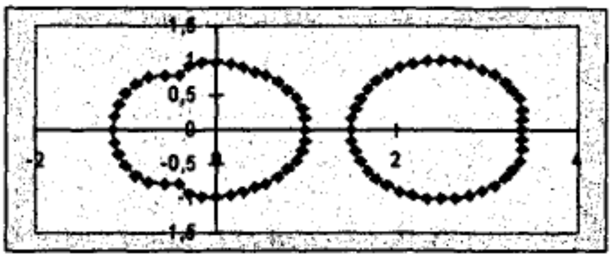

Fig. 18

On Fig. 17 the ratio of conductivities is $\sigma_{13} = 5.0$, and on Fig. 18 it is $\sigma_{13} = 10.0$ with $R_{12} = 1.0$, $E_\gamma = 0.4$, $\mu_{13} = 1.5$, $\mu_{12} = 0.5$, $\varepsilon_{21} = 2.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$, so when the drops are with equal radii and the intensity of the electric field is not strong, the change of conductivity ratio has no significant influence on the deformations.

The problem for the finite deformations of two drops due to electric field has ten dimensionless parameters, each of them having an influence on the process of deformations somehow, so further results will be presented in next papers.

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