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ON THE ANALOGY BETWEEN THE MAXWELL ELECTROMAGNETIC FIELD AND THE ELASTIC CONTINUUM

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We attribute the action at a distance in electromagnetic phenomena to the effect of the internal stresses in an absolute continuous medium called *metacontinuum*. We show that Maxwell equations are straightforward corollaries from the governing equations of an incompressible elastic medium with the shear waves corresponding to the electromagnetic waves. The main advantage of the new description is that it enables one to incorporate the nonlinearity, whose manifestations turn out to be the presence of the so-called Lorentz force, and a Galilean invariance of the model. Another generalization of the model consists in acknowledging a high-grade elasticity which introduces dispersion into the governing system. A self-similar solution is found for the plane dispersive shear waves and shown to result in red-shift even when the source of light is not moving. In order to detect the existence of an absolute continuum, a revision of the classical Michelson-Morley experiment is proposed in which the effect is of first order and is not canceled by the FitzGerald-Lorentz contraction.

Keywords: Maxwell equations, scalar and vector potentials, incompressible elastic continuum, dispersive effects in electromagnetic field, Doppler effect

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INTRODUCTION

Electromagnetic phenomenon is an epitome of an action at a distance. The latter is hard to imagine without some kind of a material carrier and the nineteen centuries tradition has been always connected with some mechanical construct. In

Cauchy's and Hamilton's vision it was a lattice whose continuum approximation yielded the elastic-body model. In Maxwell's imagination the medium had internal degrees of freedom and the action at a distance was mediated by infinitesimal "gears" transmitting the momentum from point to point. McCullagh and Sommerfeld quantified this idea as elastic body with special rotational elasticity (see [14] for an exhaustive review of these theories). After Lord Kelvin came up with the model of fluid aether and its vortex theory of matter, the coinage "aether" assumed almost exclusively fluid meaning. Paying tribute to the tradition, all of the models for the presumably absolute continuous media underlying the physical world were called "aether", although some of them (the different elastic models, for instance) were quite different from an "aetherial fluid" (where the coinage comes from).

In the 19th century, the question of entrainment of the aether (aether-drift) was perceived as pivotal and the nil result of Michelson and Morley experiment [10, 11] was understood as a blow to the whole edifice of aether theories. However, the notion of a material carrier of the long-distance interactions could not be dismissed altogether and the conceptual vacuum was filled by the concept of "physical vacuum" or the "field" which possessed all the properties of the disgraced aether, e.g., action at a distance, but it was deliberately exempted from the obligation to be checked for aether-drift effect. It was then advertised as a thing in itself not connected to any "primitive mechanistism".

In our point of view, the *field* (*physical vacuum, aether*) can only be understood from the point of view of an absolute material continuum in which the *internal stresses* are the transmitter of the long-range interactions. In order to distinguish it from the mechanical continuous media (bodies, liquids, gases, etc.), we call the continuum-mechanics model of the unified field *metacontinuum* in the sense that it is beyond (*meta*) the observable phenomena and is their progenitor.

A valid candidate for the luminiferous field is the elastic medium, because, as shown from Cauchy himself (see [14]), it gives a good quantitative prediction for the shear-wave phenomena (light). Building upon our previous work [1, 2], we show here that the Maxwell equations follow from the linearized governing equations of the metacontinuum provided that the electric and magnetic fields are properly understood as manifestation of the *meta* internal stresses. The main difference from the Cauchy's volatile elastic aether (zero dilational elastic modulus) is that we consider the opposite limiting case: an elastic continuum with infinitely large dilational modulus (virtually incompressible elastic medium).

The model proposed here should not be confused with McCullagh's model of pseudo-elastic continuum with restoring couples, which tried to explain the unusual form of Maxwell equations apparently not fitting into the picture of continuum mechanics (see [14, 7] for reference and further developments).

Our model naturally incorporates the Galilean invariance, while the Maxwell equations are not Galilean invariant and there is no feasible way to make them such.

In the end, we address the problems connected with experimental identification of the presence of an absolute continuum and propose a certain revisiting of the Michelson and Morley experiment.

1. CAUCHY VERSUS MAXWELL

We begin with the linearized equations of the elastic continuum. For small velocities the Lagrangian and Eulerian descriptions coincide and for the displacements $\boldsymbol{u}$ of a Hookean elastic medium one gets the linear vector wave equation

$$\begin{aligned} \mu_0 \frac{\partial \boldsymbol{v}}{\partial t} \equiv \mu_0 \frac{\partial^2 \boldsymbol{u}}{\partial t^2} &= \eta \Delta \boldsymbol{u} + (\lambda + \eta) \nabla (\nabla \cdot \boldsymbol{u}) \\ &\equiv -\eta \nabla \times \nabla \times \boldsymbol{u} + (\lambda + 2\eta) \nabla (\nabla \cdot \boldsymbol{u}) , \end{aligned} \tag{1.1}$$

where $\boldsymbol{u}, \boldsymbol{v}$ are the displacement and velocity vectors, respectively; η, λ are the Lamé elasticity coefficients and μ_0 is the density of metacontinuum in material (Lagrangian) coordinates. In this paper we concern ourselves with a continuous medium of constant elastic coefficients η, λ and constant density μ_0.

Equations (1.1) govern both the shear and the compression/dilation motions. The former are controlled by the shear Lamé coefficient η, while the latter — by the dilational (second) Lamé coefficient λ, and more specifically, by the sum $(\lambda + 2\eta)$. The phase speeds of propagation of the respective small disturbances are

$$\begin{gathered} c = \left(\frac{\eta}{\mu_0} \right)^{\frac{1}{2}} , \quad c_s = \left(\frac{2\eta + \lambda}{\mu_0} \right)^{\frac{1}{2}} , \\ \frac{c^2}{c_s^2} = \delta, \qquad \delta = \frac{\eta}{2\eta + \lambda} . \end{gathered} \tag{1.2}$$

Here c, c_s are the speeds of shear and compression waves, respectively. In what follows, we call them *speed of light* and *speed of sound*, respectively. To reduce the number of governing parameters, one needs to make an assumption about the second Lamé coefficient or, which is the same, about the *speed of sound*, c_s. There are two options: to consider a so-called "volatile" continuum with $c_s = 0$ or an incompressible continuum with $c_s \to \infty$. The latter is a continuum whose speed of sound is much greater than the speed of light, i.e., $\delta \ll 1$.

Here we examine the limiting case of a virtually incompressible continuum when $\lambda \gg \eta$ ($\delta \ll 1$). In this case (1.1) can be recast as follows:

$$\delta \left(c^{-2} \frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \nabla \times \nabla \times \boldsymbol{u} \right) = \nabla (\nabla \cdot \boldsymbol{u}), \tag{1.3}$$

and the displacement $\boldsymbol{u}$ can be developed into asymptotic power series with respect to δ:

$$\boldsymbol{u} = \boldsymbol{u}_0 + \delta \boldsymbol{u}_1 + \cdots \tag{1.4}$$

Introducing (1.4) into (1.3) and combining the terms with like powers, for the first two terms we obtain

$$\nabla(\nabla \boldsymbol{u}_0) = 0\,, \tag{1.5}$$

$$c^{-2}\frac{\partial^2 \boldsymbol{u}_0}{\partial t^2} + \nabla \times \nabla \times \boldsymbol{u}_0 = \nabla(\nabla \cdot \boldsymbol{u}_1)\,. \tag{1.6}$$

From (1.5) one can deduce

$$\nabla \cdot \boldsymbol{u}_0 = \text{const}, \quad \text{or} \qquad \nabla \cdot \boldsymbol{v}_0 = 0\,, \tag{1.7}$$

which is also a linear approximation to the incompressibility condition for a continuum. In the general model of nonlinear elasticity with finite deformations, the incompressibility condition is imposed on the Jacobian of transformation from material to geometrical variables, but in the first-order approximation in δ the equation (1.7) holds true.

Henceforth we omit the index '0' for the variable $\boldsymbol{u}$ without fear of confusion. We denote formally

$$\varphi \stackrel{\text{def}}{=} -(\lambda + 2\eta)\nabla \cdot \boldsymbol{u}_1, \qquad \boldsymbol{A} \stackrel{\text{def}}{=} \boldsymbol{v}_0, \tag{1.8}$$

and recast the linearized Cauchy balance (1.6) in dimensional form as follows:

$$\mu_0 \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla\varphi + \nabla \cdot \boldsymbol{\tau}, \tag{1.9}$$

where $\boldsymbol{\tau}$ is the deviator stress tensor for which the following relation is obtained from the constitutive relation (the Hooke law) for elastic body, namely:

$$\boldsymbol{\tau} = \eta\,(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) - 2\eta(\nabla \cdot \boldsymbol{u})\boldsymbol{I}, \tag{1.10}$$

where $\boldsymbol{I}$ stands for the unit tensor. For the divergence of $\boldsymbol{\tau}$ one has

$$\nabla \cdot \boldsymbol{\tau} = -\eta\,\nabla \times (\nabla \times \boldsymbol{u}) \stackrel{\text{def}}{=} \boldsymbol{E}, \tag{1.11}$$

where $\boldsymbol{E}$ stands for the vector to which the action of the purely shear part of internal stresses is actually reduced. It has the meaning of a point-wise distributed body force and we shall call it "electric force." In terms of $\boldsymbol{E}$, Eq. (1.9) recasts to

$$\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{A}}{\partial t} - \nabla\varphi\,, \tag{1.12}$$

which involves $\boldsymbol{A}$ and φ. In the same vein we define a "magnetic induction", $\boldsymbol{B}$, and "magnetic field", $\boldsymbol{H}$, as follows:

$$\boldsymbol{B} = \mu_0 \nabla \times \boldsymbol{A} = \mu_0 \boldsymbol{H}, \quad \boldsymbol{H} \stackrel{\text{def}}{=} \nabla \times \boldsymbol{A}. \tag{1.13}$$

The system of equations (1.12), (1.13) is nothing else but the equations of electrodynamics in terms of $\boldsymbol{A}$ and φ, which play respectively the role of the well-known vector and scalar potentials of the electromagnetic field. In the framework of the present approach, however, these potentials are not certain non-physical quantities introduced merely for convenience. Rather they appear to be the most natural variables: velocity and pressure of elastic luminiferous continuum. Note that the density, μ_0, of the elastic *meta*continuum appears as the magnetic permittivity of the Maxwell field.

Now one can derive the original Maxwell equations. Taking the operation *curl* of (1.12) and acknowledging (1.13), one obtains

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.14}$$

which is nothing else but the first of Maxwell equations (the Faraday law). Respectively, from eqs.(1.11), (1.13) and (1.8) one obtains

$$\frac{1}{\eta}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times (\nabla \times \frac{\partial \boldsymbol{u}}{\partial t}) \equiv \nabla \times \boldsymbol{H}\,. \tag{1.15}$$

The last equation is precisely the "second Maxwell equation" provided that the shear elastic modulus of metacontinuum is interpreted as the inverse of electric permittivity $\eta = \varepsilon_0^{-1}$. This equation has been postulated by Maxwell [9] as an improvement over the Ampere law incorporating the so-called displacement current $\partial \boldsymbol{E}/\partial t$ in the Biot-Savart form. For the case of a void space, however, when no charges or currents are present, the second Maxwell equation lives a life of its own and the Ampere law plays merely heuristic role in its derivation. It is broadly accepted now that the second Maxwell equation is verified by a number of experiments. Here we have shown that it is also a corollary of the elastic constitutive relation for the metacontinuum and is responsible for the propagation of the shear stresses (action at a distance) in *metacontinuum*.

Thus the two main (time dependent) equations of the Maxwell form have already been derived. The condition $\operatorname{div} \boldsymbol{H} = 0$ (third Maxwell equation) follows directly from the very definition of magnetic field. Similarly, taking the divergence of both sides of (1.11), one immediately obtains the fourth Maxwell equation $\operatorname{div} \boldsymbol{E} = 0$. Thus we have shown that the Maxwell equations follow from the linearized governing equations of the Hookean elastic medium whose dilational modulus is much larger than the shear one.

Thus the first objective of the present work has been achieved. We have shown that the linearized equations of elastic continuum admit what can be called *Maxwell form*. In the framework of such a paradigm, each point of the elastic continuum experiences a body force $\boldsymbol{E}$ to which the action of the internal elastic stresses is reduced. We call it "electric force."

Let us note that the governing equations of the metacontinuum are *Galilean* invariant, while the linearized version (1.1) (and hence, the Maxwell form) has lost this important property. The lack of Galilean invariance of Maxwell equations is their strangest property.

A far reaching consequence of the previous section is that it gives a clue of how to seek for a Galilean invariance of the equations of the luminiferous field (the *metacontinuum*). In the classical continua the Galilean invariance is connected to the convective (advective) nonlinearity of the governing equations. The question of what kind of effects are to be expected due to the presence of the advective nonlinear terms stands. In other words, which interactions in the metacontinuum are perceived as additional forces if the observer is limited by the concept of linear Maxwell field.

Looking for confirmation of the possible nonlinearity of the model, one stumbles into a very strange (from Maxwell's point of view) entity called "Lorentz force". Consider the governing equations of an elastic continuum in the so-called Lamb form (see, e.g., [12]):

$$\mu_0\left(\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{2}\nabla|\boldsymbol{v}|^2-\boldsymbol{v}\times\operatorname{rot}\boldsymbol{v}\right)+\nabla\varphi=-\boldsymbol{E}, \tag{2.1}$$

where we have already substituted the notations for the above defined scalar potential and electric field.

This form allows one to assess the forces acting at a given material point of the metacontinuum due to the convective accelerations of the latter. The gradient part of the convective acceleration can not be observed independently from the pressure gradient φ in the metacontinuum. In fact, one can measure only the quantity $\varphi_1 \equiv \varphi + \frac{1}{2}\boldsymbol{v}^2$. Thus the only observable effect of the nonlinearity is connected with the last term of the acceleration. By virtue of our definition of magnetic induction (1.13), the term under consideration adopts the form

$$F_l = \mu_0 \boldsymbol{v}\times \mathrm{B}. \tag{2.2}$$

Equation (2.2) expresses the force acting in each material point of the metacontinuum. This force is a part of the inertial force in the metacontinuum. In order to find its relation to the Lorentz force, one has to insert a test charge propagating through the continuum, to integrate (2.2) over the support of the charge and to get something proportional to the Lorentz force experienced by a moving charge. The exact coefficient of proportionality can be checked only after the notion of charge is incorporated into the model (see, [2]). Yet the expression (2.2) has an important bearing, because it involves the same quantities as the Lorentz force. It points out the direction in which the governing equations (1.1) of the electromagnetic dynamics can be generalized so that to become *Galilean* invariant.

The paradigm based on incompressible metacontinuum has already been instrument in restoring the Galilean invariance in the electromagnetic phenomena. However, the significance of the new concept is important in much wider field. It opens a more natural way of treating the mechanical properties of the unified field.

Let us consider now the (1+1)D-motions (plane waves). This is a good approximation for the spherical waves far from the emitting source. The governing equations for nonlinear elastic waves reduce to

$$\frac{\partial^2 w}{\partial t^2} = c^2 \left(1 + \frac{\partial w}{\partial x}\right) \frac{\partial^2 w}{\partial x^2} - \kappa \frac{\partial^4 w}{\partial x^4}, \tag{3.1}$$

where $w = u_y(x,t)$ is one of the transversal components of the vector of elastic displacements $\boldsymbol{u}$ and c is the phase speed of the shear waves (speed of light). Respectively, κ is the dispersion coefficient. It is completely natural to have some higher grade elasticity in the metacontinuum, which results in the fourth-order terms.

Consider now the evolution of the wave profile $w(x,t)$ in the frame moving with velocity c in the positive x-direction and introduce new independent coordinates and sought function:

$$t_1 = \frac{1}{2} t, \quad x_1 = x - ct, \qquad w(t,x) = w_1(t_1, x_1).$$

The different derivatives are expressed as follows:

$$\begin{gathered} \frac{\partial w}{\partial x} = \frac{\partial w_1}{\partial x_1}, \quad \frac{\partial w}{\partial t} = \frac{1}{2}\frac{\partial w_1}{\partial t_1} - c\frac{\partial w_1}{\partial x_1}, \\ \frac{\partial^2 w}{\partial t^2} = \frac{1}{4}\frac{\partial^2 w_1}{\partial t_1^2} - c\frac{\partial^2 w_1}{\partial t_1 \partial x_1} + c^2\frac{\partial^2 w_1}{\partial x_1^2}. \end{gathered} \tag{3.2}$$

If we consider only motions for which the evolution in the moving frame is very slow, then we can disregard the local time derivatives with respect to the local spatial derivatives in the sense that

$$\left| \frac{\partial^m w_1}{\partial t_1 \partial x_1^{m-1}} \right| \ll c \left| \frac{\partial^m w_1}{\partial x_1^m} \right|, \quad \left| \frac{\partial^2 w_1}{\partial t_1^2} \right| \ll c \left| \frac{\partial^2 w_1}{\partial t_1 \partial x_1} \right|.$$

Upon introducing (3.2) into eq. (3.1) and neglecting the terms according to the above scheme, we arrive at the following approximate equation:

$$-c\frac{\partial^2 w_1}{\partial x_1 \partial t_1} = \frac{\partial}{\partial x_1}\left[\frac{c^2}{2}\left(\frac{\partial w_1}{\partial x_1}\right)^2 - \kappa\frac{\partial^4 w_1}{\partial x_1^4}\right]. \tag{3.3}$$

Since the exact spatial position of the source is immaterial, one can chose it in a manner so that after one integration with respect to x_1 the integration constant to be set equal to zero. Thus eq.(3.3) recasts

$$\frac{\partial w_1}{\partial t_1} = -\frac{c}{2}\left(\frac{\partial w_1}{\partial x_1}\right)^2 + \frac{\kappa}{c}\frac{\partial^3 w_1}{\partial x_1^3}. \tag{3.4}$$

The dimension for κ is L^4T^{-2}, where L is a length scale, T is a time scale. Inverting these relations, we get $T = c^2\kappa^{-\frac{1}{2}}$ and $L = \kappa^{\frac{1}{2}}c^{-1}$. Then one can introduce dimensionless variables according to the scheme

$$w_1 = \frac{\kappa^{\frac{1}{2}}}{c}\hat{w}, \qquad t_1 = \frac{\kappa^{\frac{1}{2}}}{c^2}\hat{t}, \quad x_1 = \frac{\kappa^{\frac{1}{2}}}{c}\hat{x},$$

and to render (3.4) to the following dimensionless equation:

$$\frac{\partial \hat{w}}{\partial \hat{t}} = -\frac{1}{2}\left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2 + \frac{\partial^3 \hat{w}}{\partial \hat{x}^3}. \tag{3.5}$$

In what follows, we omit the "hats" without fear of confusion. The above equation admits a self-similar solution of the following type:

$$w = t^{-b}F(\eta), \qquad \eta = \frac{x}{t^a}. \tag{3.6}$$

To demonstrate this, we find the expression for the different terms:

$$\frac{\partial w}{\partial t} = t^{-1-b}[F(\eta) - a\eta F'(\eta)], \quad \frac{\partial w}{\partial x} = t^{-b-a}F'(\eta),$$
$$\frac{\partial^3 w}{\partial x^3} = t^{-b-3a}F'''(\eta),$$

and introduce them in (3.5) to obtain

$$t^{-1-b}[F(\eta) - a\eta F'(\eta)] = -\frac{1}{2}t^{-2b-2a}\left(F'(\eta)\right)^2 + t^{-b-3a}F'''(\eta).$$

A self-similar solution is possible only if

$$-1-b = -2b-2a, \quad -1-b = -b-3a,$$

i.e.

$$a = b = \frac{1}{3}. \tag{3.7}$$

This kind of solution has been found in [3] for the Boussinesq equation. The numerical simulation [3] has confirmed the self-similar behavior of the solution: the support increases with $t^{1/3}$, while the amplitude of the pulse decreases proportionally to $t^{-\frac{1}{3}}$.

What are the implications of the existence of a self-similar solution? The second of equations (3.6) yields that the characteristic spatial scale of the solution increases with $t^{\frac{1}{3}}$. This means that for harmonic waves the wave length will increase accordingly. Since we consider a moving frame, then the dimensionless time t is measured by the distance $r = ct$ travelled by the solution from the source. Then in our model the Hubble coefficient of proportionality between the relative red shift and the distance from the source will not be constant, but will decrease as $r^{-2/3}$. It is well-known that as the observations push to the far limits of the Universe, the Hubble constant decreases. And the controversy is still raging about the value of the Hubble "constant".

The data for the Hubble constant from a single investigation is not enough to identify the coefficient κ. It will take a more sophisticated approach compiling all the available data for all different distances. This warrants a special investigation, which goes beyond the framework of the present paper. What we can claim here is that far from the source one can not distinguish between the red-shifting due to the presence of dispersion and the one that results from a Doppler effect (if present). This means that when a dispersion is present, then the "red shift" can be alternatively explained without the help of "Big-Bang" hypothesis.

4. ABSOLUTE MEDIUM AND THE FIRST-ORDER DOPPLER EFFECT

The foremost implication of the above described relationship between the Maxwell equations and the equations of elastic continuum is that there is a place for absolute continuum. It is clear that one cannot propose anything so radical without proposing an experimental scheme to detect the absolute continuum or at least to comment on the known ones. We do not mean some more radical revision like [13], where new hypothesis about the isotropy of the speed of light has been tested. Rather we propose revisiting the classical Michelson experiment.

To use interferometry for the verification of Doppler effect has been proposed by Maxwell [8]. It was widely believed that discovering a Doppler effect would prove the existence of an absolute medium at rest. Way before the actual experiment was performed by Michelson, Maxwell pointed out that any interferometry experiment, involving splitting and reflecting the same ray, inevitably renders the sought effect of *second order* with respect to the small parameter $d = v_e c^{-1}$ (v_e stands for the velocity of Earth with respect to the quiescent medium). In the worst case $d \approx 10^{-4}$, which corresponds to the orbital velocity of Earth. If the whole solar system is moving faster through the void, then d could be larger and the effect would be more appreciable.

It was Michelson [10] who implemented such a precise experiment. And after a nil effect was observed, a prolonged discussion began about the accuracy of Michelson's experimental observations. The set-up was later on refined by Michelson and Morley [11] (MM, for brevity) and the absence of the expected type of interference was confirmed more decisively.

In our opinion, the nil effect of MM experiment cannot disprove the existence of absolute medium, because the only conclusion that can be drown strictly from the nil effect is not that the absolute medium does not exist, but rather that in the medium where the light is being propagated there occurs an apparent contraction of the spatial scales in the direction of motion of the source (FitzGerald-Lorenz contraction) proportional to the factor

$$1 - \frac{v_e^2}{2c^2} \approx \sqrt{1 - \frac{v_e^2}{c^2}}\,,$$

which exactly compensates for the expected second-order effect.

All this means that the real proof of the existence or non-existence of the *meta-continuum* can be furnished only through measuring the first-order effect. We deliberately exclude from consideration any kind of non-optical experiment and leave beyond our scope the optical experiments in dense matter (water filled columns, etc). A situation in which the first-order effect is not canceled can be created if *two different* sources of light are employed with sufficiently well synchronized frequencies. Lasers with the required level of stabilization of the frequency are available nowadays, but it goes beyond the frame of the present work to deal with the specific problems of the hardware. It suffices only to mention that if the two sources are synchronized up to 10^{-6}, the accuracy would be of order of 1% compared to the magnitude of the sought effect which is at least of order of 10^{-4}.

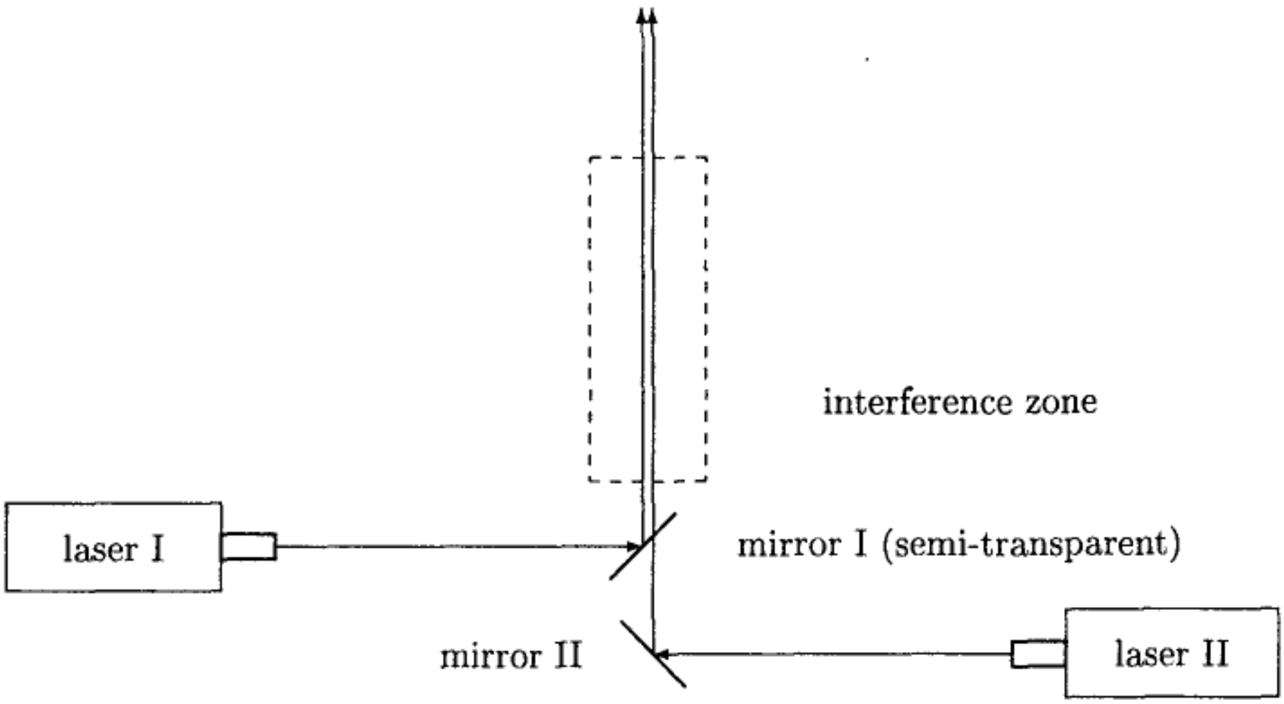

Fig. 1. Principal scheme of the interferometry experiment

In Fig. 1 the scheme of a possible first-order experiment is presented. Consider two "identical" (synchronized within 10^{-6}) sources of monochromatic light, which move together in the same direction with the same velocity. The first of them emits a plane wave propagating in the direction of motion and the second one — in the opposite direction.

By means of one mirror and one semi-transparent mirror, the two plane waves are made co-linear. The role of the mirrors is to change the direction of propagation of each wave without destroying its plane nature. The beam of the second laser is reflected by the mirror II changing its direction on 90° and making it to pass through a semi-transparent mirror whose reflecting surface serves to change the direction of the beam of the first laser on 90°. Beyond the semi-transparent mirror the two beams are parallel and can produce an easily detectable interference pattern. A snapshot of the region of interference would reveal strips of different intensity, gradually transforming into each other, so that the modulation frequency can be estimated.

It is interesting to note that Jaseda et al. [5, 6] already have used two lasers in an interferometry experiment in order to verify quantitatively the FG-L contraction, but in their experiments the lasers beams are parallel, while in the proposed here experiment they are anti-parallel since now it is not the contraction that needs verification, but the very existence of the first-order of Doppler effect.

Following [4], we outline here the derivation of the Doppler effect. The plane harmonic waves propagating in a presumably quiescent medium are given by the following formula:

$$F_{\pm}(x,t) \equiv e^{i(k_r x \mp \omega_r t)}, \quad k_{\pm} = \frac{\omega_r}{c}, \quad \kappa_{\pm} = \frac{c}{\omega_r}, \tag{4.1}$$

where $\omega_{\pm}$ are the frequencies. The upper sign in the notations refers to the wave propagating in the positive direction, while the lower sign — to the wave propagating in the negative direction. These waves have to satisfy the boundary condition on the moving boundaries (the sources):

$$F_{\pm}(\pm v_e t, t) \equiv e^{i\omega_0 t}, \tag{4.2}$$

where v_e is the velocity of the moving frame relatively to the metacontinuum. If the sources were at rest, then they would have produced waves with wave number $k_0 = \omega_0/c$ and wave length $\kappa_0 = k_0^{-1}$. The boundary condition (4.2) yields the following relation for the parameters of the propagating wave:

$$\omega_{\pm} = \omega_0 \left(1 \mp \frac{v_e}{c}\right)^{-1}, \quad k_{\pm} = \frac{\omega_0}{c}\left(1 \mp \frac{v_e}{c}\right)^{-1}, \quad \kappa_{\pm} = \kappa_0 \left(1 \mp \frac{v_e}{c}\right). \tag{4.3}$$

After the reflection, the two waves are propagating as plane waves in the positive direction of z-axis (vertical in Fig. 1: $F_{\pm}(z,t) \equiv e^{i(k_{\pm} z - \omega_{\pm} t)}$). Then in the interference region one has a wave which is the superposition of two of them for a given moment of time (say, $t = 0$):

$$\mathrm{Re}\,|F_{+}(z,t) + F_{-}(z,t)| = 2\cos\left(\frac{k_{+} + k_{-}}{2} z\right)\cos\left(\frac{k_{+} - k_{-}}{2} z\right), \tag{4.4}$$

which is a modulated wave with a wave number of the carrier $\frac{1}{2}(k_+ + k_-) = k_0 + O(d^2)$ and $\frac{1}{2}(k_+ - k_-) = dk_0 + O(d^3)$ for the modulation. Respectively, the expressions for wave lengths valid to the second order are κ_0 and $\kappa_m = \kappa_0 d^{-1}$. For $d = 10^{-4}$ and red-light lasers, the length of the wave is $\kappa_0 \approx 6.3 \cdot 10^{-5}$ cm, and then for the length of modulation wave one has $\kappa_m = 0.63$ cm and the strips produced must be easily detectable on an optical table of standard dimensions.

5. CONCLUDING REMARKS

In the present paper we have shown how the Maxwell equations can be derived from the linearized equations of an incompressible elastic continuum. Thus for the electromagnetic phenomena, we have arrived at the notion of underlying continuous medium, which we call metacontinuum. The approach of the present work admits feasible generalization to include inertial effects in the metacontinuum. This may explain the so-called Lorentz force as an effect of the advective nonlinearity of the governing equations. Moreover, incorporating the advective part of the time derivative makes the proposed model Galilean invariant, i.e. a Galilean Electrodynamics is developed.

The new concept provides a sound foundation for incorporating additional information about the electromagnetic phenomena, opening the way for pertinent generalizations. As a featuring example of such a generalization, the incorporation of dispersion (high-grade elasticity) in the governing equations is considered. A self-similar solution is found for the latter case in which the dispersion acts as to increase the spatial scale (wave-length) of the solution with the cubic root of the distance from the source of wave.

In the end, we describe the changes which are to be made in Michelson-Morley experiment in order to be able to detect a first-order Doppler effect in an absolute medium. We show that the first-order Doppler effect is not affected by the FitzGerald-Lorentz contraction.

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